BANCO CENTRAL DE COSTA RICA

DOCUMENTOS DE TRABAJO

N.º 02|2026

Allocative Efficiency in the Manufacturing Sector: A Firm-Level Analysis for Costa Rica

Claudio A. Mora-García
Melissa Vega-Monge

Fotografía de portada: "Presentes", conjunto escultórico en bronce, año 1983, del artista costarricense Fernando Calvo Sánchez. Colección del Banco Central de Costa Rica.

Eficiencia asignativa en el Sector manufacturero: Un análisis a nivel de firma para Costa Rica

Claudio A. Mora-García†

Melissa Vega-Monge‡

Resumen

Este documento analiza la evolución de la asignación eficiente de recursos (i.e., eficiencia asignativa) en el sector manufacturero de Costa Rica con datos administrativos a nivel de la firma entre 2005 y 2022. Con base en el marco estructural de Blackwood et al. (2021), estimamos la productividad y la eficiencia asignativa bajo los supuestos de retornos constantes y no constantes a escala. Además, consideramos explícitamente el papel de los márgenes, la elasticidad de la demanda y los fundamentos de la firma. Nuestros resultados muestran que, si bien la eficiencia asignativa en la manufactura se mantiene relativamente baja, presenta una tendencia creciente a lo largo del tiempo. No obstante, las ganancias potenciales de eliminar la mala asignación son significativas: transitar hacia una asignación eficiente de los recursos productivos incrementaría la productividad del sector manufacturero entre un 61% y un 89%.

Palabras clave: eficiencia asignativa, productividad, datos administrativos, Costa Rica

Clasificación JEL: D22, D24, D61, L60, O47

†Departamento de Investigación Económica. División Económica, BCCR. moragl@bccr.fi.cr.
‡Departamento de Investigación Económica. División Económica, BCCR. vegamm@bccr.fi.cr.

Allocative Efficiency in the Manufacturing Sector: A Firm-Level Analysis for Costa Rica

Claudio A. Mora-García†

Melissa Vega-Monge‡

Abstract

This paper studies the evolution of allocative efficiency (AE) in Costa Rica's manufacturing sector between 2005 and 2022, using comprehensive administrative firm-level data. Building on the structural framework of Blackwood et al. (2021), we estimate productivity and AE under both constant and non-constant returns to scale, explicitly accounting for the role of markups, demand elasticity, and firm-level fundamentals. Our results show that while AE in manufacturing remains relatively low, it exhibits an upward trend over time. Nonetheless, the potential gains from eliminating misallocation are large: moving to an efficient allocation of resources would raise manufacturing productivity by an estimated 61% – 89%.

Key words: allocative efficiency, productivity, administrative data, Costa Rica

JEL Codes:D22, D24, D61, L60, O47

†Research Deparment. Economic Division, BCCR. moragl@bccr.fi.cr.
‡Research Department. Economic Division, BCCR. vegamm@bccr.fi.cr.

1 Introduction

There has been a growing consensus among economists and policymakers regarding the importance of productivity as a critical driver of economic growth. This importance is reflected in the fact that nearly half of the differences in per capita income among countries are explained by differences in total factor productivity (TFP) (Hall and Jones, 1999). This raises a fundamental question: What accounts for these differences in TFP across countries or sectors? One possible explanation is misallocation, which refers to the inefficient distribution of productive inputs (e.g., capital and labor) across firms or sectors, such that resources are not employed where they are most productive.

Recent literature suggests that misallocation of resources across firms can affect aggregate TFP (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009, 2014; Foster et al., 2016a; Alfaro-Ureña and Garita-Garita, 2018; Blackwood et al., 2021). For instance, in environments where more productive firms face constraints that prevent them from expanding, while less productive firms continue operating inefficiently, aggregate TFP is lower than it would be under an efficient allocation benchmark (Hsieh and Klenow, 2009; Restuccia and Rogerson, 2013; Bartelsman et al., 2013). In particular, in the presence of distortions, such as credit market imperfections or regulatory barriers, resources may be misallocated. This line of research highlights that even in the absence of technological change, reallocating inputs from less productive to more productive firms can lead to substantial gains in terms of aggregate productivity (Hsieh and Klenow, 2009; Foster et al., 2016a).

Thus, analyzing the extent of resource misallocation within industries, particularly in key sectors such as manufacturing[1], provides insights into the microeconomic frictions that shape macroeconomic outcomes (Bartelsman et al., 2013; Blackwood et al., 2021). Moreover, understanding the interplay between firm-level productivity and allocative efficiency (AE) becomes particularly relevant in developing countries, where distortions in factor markets are more prevalent and institutional frameworks are less developed (Restuccia and Rogerson, 2008).

[1]Manufacturing accounted for 14.1% of value added in 2024, making it the main economic activity in Costa Rica, followed by professional, scientific and technical service activities.

The link between firm-level productivity and AE is closely related to how aggregate productivity is defined. In theory, aggregate productivity measures can be interpreted as a weighted average of firm-level productivities (Melitz and Polanec, 2015). From this perspective, aggregate productivity can increase through two different mechanisms. The first is within-firm productivity growth, which reflects improvements in technology, management, or production processes while holding the resource allocation fixed. The second is the reallocation of factors toward more productive firms. Although both channels contribute to aggregate productivity, the misallocation literature typically reserves the term AE for the between-firm reallocation channel.

Early work on AE shows that when demand is characterized by a constant elasticity of substitution (CES) and production exhibits constant returns to scale (CRS), differences in revenue per composite input, commonly labeled TFPR, reflect frictions and distortions impeding the equalization of marginal revenue products across firms (Hsieh and Klenow (2009)). More recently, Blackwood et al. (2021) derived a generalized measure of AE under non-constant returns to scale (NCRS). They showed that under NCRS, measurement of AE from revenue and input data requires decomposing revenue elasticities into output and demand components (or more broadly into returns to scale and markup components), and provides methods to do so.

Empirically, the literature quantifying AE and TFP typically relies on manufacturing surveys, as they provide the plant-level panel data necessary for this type of analysis. For example, the Annual Survey of Manufacturers and the Longitudinal Business Database are frequently used when calculating AE for the US (Blackwood et al., 2021; Hsieh and Klenow, 2009). The Colombian Annual Manufacturing Survey, the Mexican Monthly Industrial Survey, the Annual Industrial Survey, and the Chilean Annual National Industrial Survey have been used as panel-data sources for other countries (Eslava et al., 2013; Garcia-Marin and Voigtländer, 2019).

Unfortunately, plant-level observations from panel manufacturing surveys are often unavailable in many countries. In the case of Costa Rica, the analyzes are based on administrative data. Although administrative data often lack detailed information on production processes, prices, and firm organization, limiting the type of analysis that can be conducted, they offer important advantages: cover the universe of formal firms, allow for longitudinal analysis, reduce reporting bias compared to surveys, and enable the study of sectors at a more granular level.

In this paper, we discuss key considerations for calculating AE using administrative data. Following Blackwood et al. (2021), we employ both constant and non-constant returns to scale frameworks, to estimate productivity, as well as sectoral and aggregate AE in Costa Rica's manufacturing sector from 2005 to 2022. We choose the framework of Blackwood et al. (2021) because it explicitly derives the relationship between firm-level distortions and fundamentals within a structural framework that allows for the explicit treatment of NCRS, allowing firm-level distortions and fundamentals to be identified without imposing CRS. This approach is particularly useful for our analysis, as it emphasizes the importance of understanding how distortions interact with fundamentals in order to accurately measure their effects on productivity and the allocation of resources across firms.

Thus, to quantify AE, we rely on alternative measures of TFPR that capture underlying frictions, distortions, and firm-level fundamentals. Following the existing literature, we construct two measures. First, we use input cost shares of total costs, as weights to estimate a measure of revenue per composite input, resulting in a indicator we denote as $TFPR^{cs}$. As shown by Blackwood et al. (2021), $TFPR^{cs}$ reflects frictions and distortions, even when the CRS assumption is relaxed. Second, to better capture firm-level fundamentals while addressing endogeneity concerns, we apply control function methods. The revenue productivity measure derived from this approach is denoted as $TFPR^{rr}$, where "rr" stands for "revenue function residual." According to Blackwood et al. (2021), $TFPR^{rr}$ is proportional to fundamentals.

Finally, estimating AE also requires estimates of returns to scale and markups. To this end, we adopt two complementary approaches. First, we implement the approach of DeLoecker and Warzynski (2012) to estimate markups at the industry level, assuming CRS. Second, we extend our control function approach by combining it with the relationship between firm-level and industry-level variation, as in Klette and Griliches (1996), to decompose revenue elasticities into its returns to scale and markup components. An advantage of this approach is that it does not impose CRS.

By doing so, we contribute to the literature in several ways. First, recent papers examining sectoral and aggregate productivity in Costa Rica do not quantify the degree to which resources are efficiently allocated across firms and sectors (Ivankovich-Escoto and Martínez-Castillo, 2020; Monge-González, 2019; Monge-González et al., 2020; Robles, 2021; Vega-Monge and Jiménez-Montero, 2025). On the other hand, the most recent study on AE in Costa Rica was conducted by Alfaro-Ureña and Garita-Garita (2018). The authors assess the extent of resource misallocation and the potential productivity gains from removing distortions, estimating theoretical gains of approximately 50%-60%. While their study provides valuable insights into the relationship between productivity and distortions in Costa Rica, the empirical approach chosen by them present some limitations, including the fact that some key parameters, such as the elasticity of substitution, are borrowed from the literature. Moreover, it is important to note that the study relies on data from 2005 to 2015, which may not fully capture the current dynamics and changes of Costa Rican economy.

Thus, replicating the framework of Blackwood et al. (2021) and Hsieh and Klenow (2009) in a different economic context, such as Costa Rica, contributes to the broader literature by testing the robustness and applicability of their methodology in a small open economy with sustained productivity growth over the past decade. This comparison deepens our understanding of the mechanisms behind productivity growth and clarifies the role of AE in shaping economic performance.

Four main findings emerge from this study. First, our analysis shows that, despite differences across estimation methods, productivity measures are highly correlated, reinforcing their robustness for studying productivity and misallocation. Second, firm-level patterns, such as the positive relationship between productivity, growth, and survival, are consistent across approaches. Third, the analysis highlights the importance of accounting for sectoral heterogeneity in both demand and production structures when estimating AE. In particular, changes in demand elasticity estimates and returns to scale assumptions affect AE levels. Fourth, while the overall level of AE remains relatively low, the evidence points to an upward trend over time. However, the potential productivity gains from improving resource allocation remain substantial. We find that moving to the efficient allocation would increase productivity by 61% - 89% in the manufacturing sector.

The remainder of the paper is organized as follows. Section 2 presents the theoretical framework, Section 3 describes the data, Section 4 discusses the empirical results, and Section 5 concludes.

2 Methodology

2.1 Revenue Productivity Measures

This section closely follows the methodology and notation of Blackwood et al. (2021), which develop a generalized measure of AE under both CRS and NCRS. The model assumes a Cobb-Douglas production function and a CES demand structure, which are common assumptions in the literature (Hsieh and Klenow, 2009; Bartelsman et al., 2013; Foster et al., 2016b; Bils et al., 2021; Blackwood et al., 2021). Industry output is a CES aggregate of intermediate goods producers given by $Q = \left(\sum_i (\xi_i Q_i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$, where σ is the elasticity of substitution, ξ_i denotes an idiosyncratic demand shifter for plant i, Q_i denotes plant-level quantity, and Q industry level quantity. The inverse demand function is given by $P_i = PQ^{1/\sigma}Q_i^{-1/\sigma}\xi_i^{\frac{\sigma-1}{\sigma}}$ for plant i in an industry where P_i denotes plant-level prices and P industry level prices. We write P_i as $P_i = PQ^{1-\rho}Q_i^{\rho-1}\xi_i^{\rho}$, where $\rho = \frac{\sigma-1}{\sigma}$ with $0 < \rho < 1$. Note that $1/\rho$ represents the markup over marginal costs [2].

The plant-level production function is given by $Q_i = \mathcal{A}_i \prod_j X_{ij}^{\alpha_j}$, where $\mathcal{A}_i$ is technical efficiency, X_{ij} are plant-level factor inputs (e.g., capital, labor, and intermediate inputs), and α_j is the output elasticity with respect to the jth input X_{ij}. We denote returns to scale as $\gamma_i = \sum_{ij} \alpha_j$, where CRS corresponds to $\gamma_i = 1$. The log of the revenue function is given by:

$$\log P_i + \log Q_i = \sum_j \beta_j \log X_{ij} + \rho \log \mathcal{A}_i + \rho \log \xi_i + (1-\rho) \log Q + \log P, \tag{1}$$

where the revenue elasticities satisfy $\beta_j = \rho\alpha_j$. Various revenue productivity measures have been used in the theoretical and empirical literature. One typical measure is logTFPR, given by (Foster et al., 2008):

$$\log TFPR_i = \log P_i + \log Q_i - \sum_j \alpha_j \log X_{ij} = \log P_i + \log \mathcal{A}_i, \tag{2}$$

As in Blackwood et al. (2021) and much of the related literature, we face data limitations that prevent decomposing $TFPR_i$ into its price and technical efficiency components, since the available micro-level data contain information on revenues and costs, but not on plant-level

[2] Time and industry subscripts are omitted to simplify the presentation and focus on the core mechanisms of the model.

prices. This common constraint motivates our dependence on revenue-based productivity measures. However, note that under the additional assumptions that plants minimize total costs and operate under CRS, the share of the jth input expenditure in total costs equals α_j. Formally:

$$\log TFPR_i^{cs} = \log P_i + \log Q_i - \sum_j cs_j \log X_{ij} = \log TFPR_i + \sum_j (\alpha_j - cs_j) \log X_{ij}, \quad (3)$$

where cs_j denotes the cost share of the jth input. Note the equivalence between $\log TFPR_i$ from equation (2) and $\log TFPR_i^{cs}$ does not hold without CRS. Blackwood et al. (2021) show that $\log TFPR_i^{cs}$ is of interest in and of itself, even without CRS, since it is indicative of distortions under certain assumptions.

Following Blackwood et al. (2021), the revenue productivity measures discussed above differ from the revenue function residual, defined as:

$$\log TFPR_i^{rr} = \log P_i + \log Q_i - \sum_j \beta_j \log X_{ij} = \rho \log \mathcal{A}_i + \rho \log \xi_i + (1-\rho) \log Q + \log P \quad (4)$$

This expression highlights that the revenue function residual captures firm-level technical efficiency and idiosyncratic demand shocks, while also reflecting common aggregate components such as prices and quantities. This distinction between equations (3) and (4) is central for separating the role of fundamentals and distortions.

Following Hsieh and Klenow (2009) and subsequent literature, in the absence of idiosyncratic frictions or distortions, marginal revenue products would be equalized across production units, implying no within-industry dispersion in $\log TFPR_j^{cs}$. Because such equalization is not observed in the data, this counterfactual benchmark motivates the introduction of firm-level distortions to explain the empirically observed dispersion in revenue productivity.

A key finding from Blackwood et al. (2021) is that $TFPR_i^{cs}$ is proportional to idiosyncratic distortions τ_i while $TFPR_i^{rr}$ is proportional to fundamentals $\mathcal{A}_i\xi_i$. They show that the decision problem of firms that maximize static profits with input distortions implies[3]:

$$TFPR_i^{cs} \propto \tau_i \tag{5}$$

where $\tau_i = \prod_j \left(1+\tau_{ij}\right)^{\alpha_j/\gamma}$ denotes a plant-specific weighted geometric average of input distortions and the weights are given by cost shares.

In addition, Blackwood et al. (2021) shows that $TFPR_i^{rr}$ is proportional to plant technical efficiency and demand shocks under the same assumptions, while abstracting from industry-level shifters that can be captured by industry-year effects:[4]

$$TFPR_i^{rr} \propto \left(\mathcal{A}_i \xi_i\right)^{\rho}. \tag{6}$$

Thus, this conceptual difference is what motivates Blackwood et al. (2021) analysis.

Under the core assumptions, there is no systematic relationship between $\log TFPR_i^{cs}$ and $\log TFPR_i^{rr}$. However, in practice, a variety of mechanisms, such as financial constraints, and variable markups, may covary with firm fundamentals[5]. Thus, a key contribution of Blackwood et al. (2021) is to study the systematic relationship between these conceptually distinct measures of productivity, while recognizing that a limitation of this approach is that the underlying sources cannot be separately identified, as the wedges identified in this framework capture in reduced form all forces that impede the equalization of marginal revenue products.

2.2 Allocative Efficiency

Under the core assumptions made in the prior section and assuming that all production factor supplies are exogenous (which is the case we explore in this paper), Blackwood et al. (2021) show that sectoral AE can be expressed as a function of sectoral fundamentals and distortions

[3]Following Hsieh and Klenow (2009) and Blackwood et al. (2021), the profit function in this case is given by $P_i Q_i - \sum_j w(1+\tau_{ij})X_{ij}$ where w_j denotes the jth input price. To obtain equation (5), notice that $TFPR_i = P_i \mathcal{A}_i$. It can be shown that, under the core assumptions, $TFPR_i = (1/\rho)\prod_j (MRPX_{ij}/\alpha_j)^{\alpha_j}$ where $MRPX_{ij} = \rho P_i Q_i \alpha_j / X_{ij}$ is the marginal revenue product of input X_{ij}. This means that $TFPR_i$ is proportional to the marginal revenue products of the inputs. Profit maximization implies that $MRPX_{ij} = w_j(1+\tau_{ij})$, where τ_i are idiosyncratic input distortions–without such distortions, marginal revenue products are equalized across production units and there would be no within-industry dispersion in $TFPR_i$. Substituting yields that $TFPR_i = (1/\rho)\prod_j \left(w(1+\tau_{ij})/\alpha_j\right)^{\alpha_j}$, so $TFPR_i \propto \prod_j (1+\tau_{ij})^{\alpha_j} = \tau_i$.

[4]By definition, the revenue function residual is given by $\log TFPR_i^{rr} = \log P_i + \log Q_i - \sum_j \beta_j \log X_{ij} = \rho \log \mathcal{A}_i + \rho \log \xi_i + (1-\rho)\log Q + \log P$, which implies that $TFPR_i^{rr} \propto (\mathcal{A}_i \xi_i)^{\rho}$. Where β_j are the revenue elasticities.

[5]As noted by Blackwood et al. (2021), dispersion in markups across producers generates idiosyncratic wedges. Allowing markups to vary systematically with producer scale is a standard approach in the productivity and misallocation literature.

(see Appendix A):

$$AE_s^{COV} = \left(\frac{1}{N_s} \sum_i \left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}}. \quad (7)$$

where $A_{is} = \mathcal{A}_{is}\xi_{is}$ collapses the combined effect of demand shifts and technical efficiency for notation convenience, N_s is the number of firms in the sector, the power mean of A_{is} is given by $\tilde{A}_s = \left(N_s^{-1}\sum_i A_{is}^{\rho_s/(1-\rho_s\gamma_s)}\right)^{(1-\rho_s\gamma_s)/\rho_s}$, and $\tilde{\tau}_s$ is the harmonic revenue weighted mean of distortions.[6] The equation above assumes that $\rho_s\gamma_s < 1$, which is economically intuitive, since an equilibrium with increasing returns in the revenue function would imply market dominance by a single firm.

The prior literature has labeled this term as AE_s^{COV} to emphasize that it resembles a covariance term. This expression fully accounts for the effects of NCRS. Blackwood et al. (2021) formalize this result by expressing AE_s as a function of the covariance between transformations of τ_{is} and A_{is}:

$$\log AE_s^{COV} = \gamma_s \log\left(\frac{\tilde{\tau}_s}{\bar{\tau}_s}\right) + \frac{1-\rho_s\gamma_s}{\rho_s}\log\left[\text{cov}\left(\left(\frac{A_{is}}{\tilde{A}_s}\right)^{\frac{\rho_s}{1-\rho_s\gamma_s}}, \left(\frac{\tau_{is}}{\bar{\tau}_s}\right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}}\right) + 1\right] \quad (8)$$

where $\bar{\tau}_s = \left(N_s^{-1}\sum_i \tau_{is}^{\rho_s\gamma_s/(\rho_s\gamma_s-1)}\right)^{(\rho_s\gamma_s-1)/\rho_s\gamma_s}$ is the power mean of sectoral distortions (Appendix A describes the derivation of this equation).

Equation (8) shows that AE_s^{COV} is shaped by two terms. The first term reflects the average level of distortions within the sector, while the second term captures how distortions interact with fundamentals through the covariance between exponentiated relative technical efficiencies and distortions (term 2). This second component depends on both the dispersion of these two variables, and by the sign and strength of their correlation. Studies of AE tend to focus on the dynamics of this second component, as it reflects how changes in the dispersion of fundamentals and distortions, as well as their correlation, shape AE over time. For example, if distortions are positively correlated with fundamentals, then this component of AE increases as dispersion in either A_{is} or τ_{is} declines.

[6] $\tilde{\tau}_s = S_1/S_2$ where $S_1 = \sum_i \left(A_{is}/\tau_{is}^{\gamma_s}\right)^{\rho_s/(1-\rho_s\gamma_s)}$ and $S_2 = \sum_i \left(A_{is}^{\rho_s}/\tau_{is}\right)^{1/(1-\rho_s\gamma_s)}$.

Equation (8) shows that AE depends asymmetrically on ρ_s and γ_s, for example, through their effects on the covariance between the empirical counterparts of A_{is} and τ_{is}. This motivates using alternative estimates of ρ_s and γ_s in the empirical analysis.

The crucial connection between $\log AE_s^{COV}$, $\log TFPR_{is}^{cs}$, and $\log TFPR_{is}^{rr}$ is that $\tau_{is} \propto TFPR_{is}^{cs}$ and $\log A_{is} = \frac{1}{\rho_s} \log TFPR_{is}^{rr}$. Hence, the empirical equivalent of equation (8) can be re-written as:

$$\log AE_s^{COV} = \gamma_s \log \frac{\widetilde{TFPR}_s^{cs}}{\overline{TFPR}_s^{cs}} + \frac{1-\rho_s\gamma_s}{\rho_s} \log \left[\text{cov} \left(\left(\frac{TFPR_{is}^{rr}}{\widetilde{TFPR}_s^{rr}} \right)^{\frac{1}{1-\rho_s\gamma_s}}, \left(\frac{TFPR_{is}^{cs}}{\overline{TFPR}_s^{cs}} \right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right) + 1 \right]. \tag{9}$$

Thus, measured AE_s^{COV} can be expressed as a function of the two revenue productivities measures discussed above, as well as the demand elasticity and the returns to the scale parameters (ρ_s, γ_s). Our empirical analysis below is grounded in this equation, and the next section describes how we obtain empirical estimates of each of the parameters and the corresponding TFPR measures.

Throughout the paper, we treat the supply of all production factors fixed and exogenous, and assume a Cobb-Douglas CRS aggregator for output across sectors into a final good. This method additionally assumes a perfectly competitive representative firm that produces this final output. Under these assumptions, sectoral AE simplifies to AE_s^{COV}, and overall AE is given by:

$$AE = \prod_s^S AE_s^{\theta_s} = \prod_s^S (AE_s^{COV})^{\theta_s} \tag{10}$$

where θ_s denotes the revenue share of industry s.

2.3 Estimation Methods

To quantify productivity and AE, our empirical strategy requires estimating three sets of measures: firm-level revenue productivity measures ($TFPR_{is}^{cs}$ and $TFPR_{is}^{rr}$), industry-level demand elasticity parameter and industry-level returns to scale parameter. Moreover, under certain assumptions, we can also recover output elasticities. This section describes how we obtain these ingredients. We first outline how we estimate cost-share based elasticities used to construct $TFPR_{is}^{cs}$. We then describe our control-function approach to estimating revenue elasticities and $TFPR_{is}^{rr}$. Finally, we explain how we recover the demand elasiticity parameter (ρ_s) using two complementary methods, and the returns to scale parameter (γ_s). Together, these steps provide the full set of inputs needed to implement the AE measure derived in the previous section.

We first describe the cost-share (CS) method used to construct $TFPR_{is}^{cs}$. This method relies on the first-order conditions of firm's cost minimization problem, which under CRS imply that input cost shares identify output elasticities. Input cost shares are defined as the share of input expenditures to total cost, and total costs are defined as the sum of expenditures on intermediate inputs, labor, and capital. We follow the literature and compute cost shares by aggregating expenditures across firms within industries and over time (Syverson, 2011; Blackwood et al., 2021), implicitly assuming homogeneous elasticities[7].

To estimate the revenue elasticities (β_j) and $TFPR_{is}^{rr}$, we use a control function approach. But contrary to Blackwood et al. (2021) and without data on firm-level investment in Costa Rica, we use the method proposed by Levinsohn and Petrin (2003) to address the endogeneity of unobserved productivity and inputs (hereafter LP approach). LP define conditions and assumptions under which the optimal demand for intermediate inputs (e.g., materials, electricity, fuel) can be inverted to make (unobserved) productivity a function of observable variables. From an empirical perspective, the main advantage of this method is that intermediate input data are usually available in more databases than information on firms' investments. Following LP, we use a nonparametric representation of the inverse of optimal demand to perform the estimation. We estimate industry-specific elasticities for intermediate inputs, labor, and capital using the control function approach. Our measure of intermediate inputs is an aggregate variable of input costs, as a proxy for materials.

Identifying the demand parameter ρ_s is particularly challenging, and arguably one of the most difficult components of the empirical framework to pin down. Following Blackwood et al. (2021), and recognizing that AE measures are sensitive to this parameter, our goal is not to rely on a single estimate of ρ_s. Instead, we assess the robustness of our results by using two alternative approaches to estimating ρ_s.

[7]Specifically, we add input expenditures and total costs of all firms within an industry and for all time periods, then calculate the share of each input expenditure to total costs.

The first approach follows DeLoecker and Warzynski (2012). Under the assumption of CRS, the first-order condition for a variable factor, such as intermediate inputs, implies that the markup ($1/\rho_s$) is equal to the ratio of the output elasticity to the cost share of the revenue of the variable factor. To estimate the output elasticity, we use the CS methodology and intermediate inputs at the industry level as a variable factor. We refer to this specification as DW. This method yields estimates of markups (and thus ρ_s) that vary across industries but requires the strong assumptions, including CRS.[8]

The second approach combines the control-function framework of Levinsohn and Petrin (2003) with the method of Klette and Griliches (1996), and is labeled LP–KG. In addition to estimating revenue elasticities, this approach identifies ρ_s using variation in industry-level real output, as shown in equation 4. The LP–KG specification allows demand and revenue elasticities to be recovered simultaneously within a unified framework without imposing CRS.

Both approaches involve trade-offs. The DW method is simple to implement but relies on strong assumptions, including CRS. The LP–KG approach relaxes these assumptions and jointly estimates demand and revenue elasticities, but requires additional variation at the industry level. We therefore use both methods to assess the sensitivity of our results to alternative estimates of ρ_s.

Finally, under the NCRS framework, returns to scale (γ_s) and output elasticities (α_s) are obtained by scaling the estimated revenue elasticities by the demand parameter ρ_s. Thus, returns to scale are then computed as the sum of these output elasticities across inputs at the industry level, as shown in equation 1.

3 Data

3.1 Data Sources

We use the Economic Variables Registry (i.e., Revec) compiled by the Central Bank of Costa Rica for the period 2005–2022. The dataset contains information on the production units in

[8] Under CRS and the CS methodology, the output elasticity of an input X is equal to the share of input X expenditures to total costs. Under DeLoecker and Warzynski (2012) methodology, the markup is equal to the output elasticity of input X divided by the share of expenditures on input X in total sales. Hence, if we use the CS output elasticity, then the markup is simply the rate of total sales to total costs.

Costa Rica and is used as a key input for the construction of economic indicators and for empirical research. It consists of firm-level data, which incorporate annual corporate income tax returns and employer-employee records (see also, Alfaro-Ureña et al. (2022)), offering a comprehensive view of firm activity. Since part of the data are derived from administrative tax records, the unit of observation corresponds to the firm level rather than the plant level.

Specifically, we use an anonymized panel that contains annual information on sales, input costs, assets, wages, employment, and economic activity (classified according to ISIC-4). The panel covers nearly the entire population of formal firms in Costa Rica. The final dataset is consolidated at the firm-group level,[9] and all values are expressed in nominal terms.

Estimating revenue productivity (specifically, identifying revenue elasticities) requires data on revenue, capital, labor, and intermediate inputs at the firm level. Revenue is measured by total annual sales, labor is proxied by the wage bill, and capital is approximated by net assets. Input costs are used as a proxy for intermediate inputs. To express values in real terms, total sales and input costs are deflated using the implicit GDP price deflator for the manufacturing sector, while capital is adjusted using the implicit price deflator for gross capital formation.[10]

In addition, we use the dataset to construct measures of input expenditures for the estimation of cost shares. Following Alfaro-Ureña and Garita-Garita (2018), we assume a 10% cost of capital, with capital defined as previously described. Labor costs are approximated by the firm's total wage bill, which is adjusted to account for additional labor-related costs.[11] The wage bill is deflated using the Consumer Price Index (CPI). Input costs refers to changes in inventories and purchases of raw materials.

As noted earlier, a fundamental challenge in this literature is the limited availability of data, especially for multi-product firms, due to the need for detailed product-level prices and physical quantities produced. Given this limitation, and under the methodological framework proposed

[9]A single firm may report under multiple identification numbers (e.g., assigning employees to one ID and sales to another). When these IDs share ownership and operate as a single decision-making unit, they are aggregated into firm groups. Throughout the analysis, we refer to these entities simply as firms.

[10]We use the CPI from 2020 and proportionally adjust it to reflect an index where 2017 = 100.

[11]In the Costa Rican context, there is a statutory bonus equivalent to one-twelfth of the employee's annual earnings, paid each December. Along with employer contributions to the social security system, it constitutes an important component of total labor compensation. The adjustment includes both components.

by this paper, sales data provide a practical alternative that enables us to recover productivity and AE measures, thereby facilitating the analysis of firm-level productivity, despite the absence of direct quantity data.

3.2 Sample

We restrict our analysis to formal, for-profit private firms operating within the manufacturing sector. The initial sample includes approximately 51,752 firm-year observations between 2005 and 2022 in the manufacturing sector.[12]

For the empirical analysis, we restrict the sample to firms with strictly positive values for all variables needed to estimate the production function and at least two workers reported each year. Following previous work using administrative data (Banco Central de Chile, 2017; Aguirre et al., 2021), firms with extreme values, those above the 99th percentile, in the growth of net assets or revenue, the net assets-to-revenue ratio, and the workers-to-revenue ratio are excluded.

We restrict attention to industries defined at a sufficiently narrow level to reasonably assume that production elasticities are homogeneous across firms within each industry. This restriction is important for the validity of both the CS and control-function approaches, which rely on common elasticities within industries to identify productivity and AE. By working with narrowly defined industries, we reduce heterogeneity in production technologies and demand conditions that could otherwise bias the estimation. Hence, we choose a 4-digit ISIC grid (UNDP, 2008). Moreover, the number of firm-year observations within each industry should be large enough that elasticities can be estimated by LP.[13] We use all industries with more than 100 observations in it and where no single firm represents more than 60% of the sales.

According to ISIC rev 4, there are 112 such manufacturing industries of which we work with 37, which represents about 62% of total sales in the manufacturing sector between 2005 and 2022. Revec already includes an ISIC Rev. 4 classification that standardizes previous industry classification systems over time. Firms with multiple CIIU4 codes are classified under the

[12]Given the size of the Costa Rican economy, this represents approximately 2% of the sample size in Blackwood et al. (2021).
[13]LP uses high-order polynomials making estimates sensitive to small samples.

industry contributing the most to their value added.

4 Results

4.1 Elasticity Distributions

Figure 1 shows the distribution of the estimated elasticity of output (CS) and revenue (LP, LP-KG) for our three inputs: capital, labor, and intermediate inputs. The results reveal notable differences in both their location and shape. For intermediate inputs (Figure 1, Panel A), the CS-based output elasticities are generally higher than the revenue elasticities obtained through LP and LP-KG.

Panels B and C of Figure 1 show that CS-based capital and labor elasticities tend to be smaller than LP-based estimates (LP, LP-KG). Output and revenue elasticities for labor are slightly larger than those for capital, which is similar to Blackwood et al. (2021). As shown in Table 1, the average output elasticities are 0.73, 0.10, and 0.17 under CS, respectively, and the average revenue elasticities for intermediate inputs, capital, and labor are 0.54, 0.14, and 0.23 under LP, and 0.54, 0.14, and 0.23 under LP-KG. On average, the capital-to-labor ratio remains approximately 1:2, indicating that these industries are labor-intensive.

Overall, revenue elasticities under LP and LP-KG are very similar. Furthermore, these estimates are relatively consistent with those reported by Vega-Monge and Jiménez-Montero (2023) for Costa Rica between 2005 and 2021. The authors estimated a production function using the LP methodology, employing the same inputs as our study. Although not entirely comparable, since their estimation includes all sectors of the economy, they also find revenue elasticities for intermediate inputs that are of a similar magnitude to ours (0.51), but slightly larger for capital and labor (0.19 and 0.32, respectively).

Figure 2 presents the estimated values of the revenue function curvature, demand elasticity, and the corresponding implied returns to scale. At the industry level, the curvature of the revenue function quantifies how responsive revenue is to variations in output, jointly determined by the demand elasticity and the returns to scale. The similarity in revenue elasticities derived from the LP and LP-KG methods leads to closely aligned distributions of revenue curvature.

Panel A of Figure 2 illustrates the distribution of revenue curvature under both LP and LP-KG, with sample means of 0.91 and 0.90, and standard deviations of 0.07 and 0.08, respectively. These findings are consistent with existing literature, where the average industry-level revenue curvature is typically estimated to be close to one (Olley and Pakes, 1996; Klette and Griliches, 1996; Levinsohn and Petrin, 2003; Blackwood et al., 2021).

To characterize the curvature of the revenue function, we decompose it into demand elasticity (ρ_s) and returns to scale (γ_s), considering two alternative distributions for ρ_s. One distribution, based on DeLoecker and Warzynski (2012) and denoted DW, calibrates ρ_s using industry-level data under the maintained assumption of CRS ($\gamma_s = 1$). Because this restriction is not imposed in the LP-KG framework, we depart from it in this exercise and instead condition on ρ_s as exogenous (Blackwood et al., 2021). This approach allows us to recover the implied γ_s from $\sum_j \beta_{js}$.The second ρ_s distribution, denoted by LP-KG, ρ_s is estimated jointly with β_{js} following Klette and Griliches (1996). Panel B of Figure 2 shows the two distributions. Overall, both methods exhibit substantial cross-industry variation while displaying similar distributional patterns. The average demand elasticity estimate based on DW is 0.87, compared with 0.89 under LP-KG. These values are broadly consistent with estimates reported for the U.S. manufacturing sector by Blackwood et al. (2021), which fall within a range of 0.80 to 0.94.

Panel C reports the returns to scale, denoted by γ_s, implied by the estimated demand elasticities.[14] The average returns to scale are 1.04 under LP and 1.03 under KG. However, substantial heterogeneity emerges across industries, underscoring the importance of accounting for industry-specific production characteristics when evaluating firm behavior and market structure.

To help preserve well-behaved optimization problems, it is typical to assume that $\rho_s \gamma_s \leq 1$. We find that across methodologies, 93% of the industries meet such criteria.

[14]Note that the sum of LP-based revenue elasticities can be expressed as $\sum_j \beta_{js} = \rho_s \sum_j \alpha_{js} = \rho_s \gamma_s$, where ρ_s is the elasticity of demand and α_{js} are the output elasticities.

4.2 What Are the Implications of Alternative Elasticity Distributions?

In this subsection, we explore how differences in elasticity distributions shape key empirical outcomes. Specifically, we analyze dispersion of productivity across firms, the correlation patterns between alternative productivity measures, and the relationship between productivity, firm growth, and survival. By doing so, we assess the extent to which measurement choices influence conclusions about productivity dynamics.

4.2.1 Implications in Productivity Dispersion and Correlations

A natural starting point is to assess whether the choice between output elasticities and revenue elasticities materially affects the measurement of productivity dispersion. The first column of Table 2 reports the interquartile range (IQR), indicating that the average productivity gap between firms at the 75th and 25th percentiles within the typical industry lies between 0.32 and 0.44. These magnitudes are similar across methods, suggesting that dispersion patterns are broadly stable to the elasticity specification.

This conclusion is reinforced when dispersion is measured using the standard deviation, reported in the second column of the Table 2. The qualitative results remain unchanged, further supporting the robustness of productivity dispersion to alternative elasticity measures.[15]

Taken together, these findings suggest that differences in estimated elasticities have limited implications for aggregate measures of productivity dispersion and, by extension, limited effect on broad measures of misallocation. Nevertheless, while dispersion appears stable at the aggregate level, the choice of elasticity measure may still affect the interpretation of misallocation at a more granular level.

We next investigate whether different estimation methods generate similar underlying performance by examining the correlation between alternative measures of productivity. The Pearson and Spearman correlations in Table 3 indicate that the association between $\log TFPR_{is}^{rr}$ and $\log TFPR_{is}^{cs}$ is about 70%, which means that both measures exhibit similar dispersion

[15] We construct confidence intervals for both dispersion measures using bootstrapped standard errors (1,000 replications). Although not reported here, the results indicate that differences in dispersion are not statistically significant when using the standard deviation. For the interquartile range (IQR), however, dispersion under the CS method is lower than under both LP and LP-KG.

and are strongly correlated. As pointed by Blackwood et al. (2021), under the assumption of isoleastic (CES) demand, $TFPR^{rr}_{is}$ is a measure of fundamentals. Our results imply that $TFPR^{cs}_{is}$ is positively correlated with, and similarly dispersed as, fundamentals.

4.2.2 Implications in Growth and Survival

We next assess whether alternative productivity measures yield predictions consistent with standard models of firm dynamics. Following Blackwood et al. (2021), in a model with employment as the single production factor subject to adjustment frictions, incumbent firms face two key state variables each period: their lagged employment level and the contemporaneous realization of productivity. These models imply a clear selection mechanism: conditional on prior employment, firms that draw low productivity shocks are more likely to exit, whereas those experiencing higher productivity realizations tend to expand.

To evaluate whether the empirical relationship between productivity, firm growth, and exit aligns with these theoretical predictions, and whether it is sensitive to the choice of productivity measure, we estimate the following specification:

$$y_{it+1} = b_1 \omega_{it} + b_2 \theta_{size_{it}} + \boldsymbol{x'_{it}\delta} + \varepsilon_{it+1} \tag{11}$$

where y_{it+1} denotes an outcome of firm i (growth between periods t and $t+1$ or exit), ω is firm-level productivity in logs, $\theta_{size_{it}}$ is the control for initial size (log employment) in the period, and $\boldsymbol{x_{it}}$ is a vector of additional controls: firm age and full interactions of industry and year effects.

Panel A of Table 4 reports the estimated coefficient $\hat{b}_1$ for each productivity measure. A one percent increase in productivity leads to a 0.05 percentage point increase in growth under the CS, and a 0.10 percentage point increase under both LP and LP-KG; all of these effects are statistically significant at the 1% level. Regarding exit, the CS yields a statistically insignificant reduction in the probability of exit (-0.01), whereas LP and LP-KG show a significant reduction of -0.02. Conditional growth follows a similar pattern: positive and significant productivity effects across all methods, with CS at 0.05 and LP/LP-KG at 0.10.

Panel B of Table 4 shows the effects of initial firm size. Larger firms are less likely to exit across all especifications, with a consistent and statistically significant effect of -0.02. However, conditional on survival, larger firms grow more slowly: log size is associated with a -0.01 reduction in growth under CS, and -0.03 under LP and LP-KG. All size effects are statistically significant at the 1% level.

Overall, the findings in Sections 4.2.1 and 4.2.2 confirm that either estimator can be informative for understanding firm dynamics in terms of dispersion, growth, and exit behavior. Importantly, while the magnitudes of coefficients differ somewhat across estimators, especially between CS and LP/LP-KG, the overall directional implications remain stable.

We now turn to an analysis of AE, a dimension where methodological choices, both in terms of estimation approach and underlying assumption, may have significant implications.

4.3 Empirical Results on AE

This section presents the results of AE estimation under CRS (CS method) and NCRS (LP method), while also exploring the sensitivity of AE to a range of revenue curvature estimates ($\rho_s \gamma_s$). To assess this sensitivity, we examine AE using our CS, LP, and LP-KG productivity estimates across a range of demand elasticity values ρ_s. Specifically, we use $\rho_s = (0.75, 0.85, 0.88, \rho_s^{DW}, \rho_s^{KG})$. The value $\rho_s = 0.75$ corresponds to the calibration used in Hsieh and Klenow (2009) and more recently adopted by Bils et al. (2021). The values $\rho_s = 0.85$ and $\rho_s = 0.88$ represent the average demand elasticity across industries estimated using the DW and KG methods, respectively. Finally, ρ_s^{DW} and ρ_s^{KG} refer to the distributions of demand elasticities derived from the DW and KG approaches. The purpose of this exercise is to systematically assess how the choice of demand elasticities affects AE. This allows us to evaluate the robustness of AE conclusions to plausible variation in this critical parameter.

Importantly, CS and LP methods differ in their underlying assumptions regarding the estimation of the revenue curvature, which results in different implications. Under CS, $\gamma_s = 1$ is posited and, therefore, the revenue curvature is fully determined by changing the value of the demand elasticity ρ_s. Thus, the implications of different ρ_s values under CS provide guidance about the effect of changes in curvature on AE when returns to scale are fixed.

In constrast, under LP method, we can evaluate the impact of decomposing the revenue curvature into its components. In this case, revenue curvature is based on the sum of estimated revenue elasticities as described above. For example, for a given set of revenue elasticities, an increase in ρ_s implies an equivalent decline in γ_s.

In line with Blackwood et al. (2021), we perform a sensitivity analysis of AE with respect to various plausible combinations of markups and returns to scale. These combinations are drawn from the range of estimates produced by different empirical methods. In doing so, our objective is to assess the robustness of our findings and to better understand how variations in key parameters influence the measurement of AE.

Moreover, given that our analysis is based on a relatively small sample compared to other studies in the literature (Hsieh and Klenow, 2009; Blackwood et al., 2021), evaluating the robustness of our results is crucial. Exploring how AE responds to alternative parameter values allows us to assess the stability of our conclusions and to better understand the extent to which our findings may be influenced by sample-specific characteristics or methodological choices.

Figure 3 shows the cross-industry weighted average of AE_s^{COV} where the weights reflect each industry's share of total revenue. The estimates are calculated using the range of demand elasticity estimates listed above. To mitigate the influence of short-term volatility in our empirical results, we calculate time series averages of AE for three periods (2006-2010, 2011-2015, 2016-2022). The most recent period (2016–2022) is especially significant, as previous studies have documented a gradual increase in productivity in the manufacturing sector since 2015 (Vega-Monge and Jiménez-Montero, 2023).

Several observations arise from the results. First, focusing on the CS estimates under the assumption of CRS, we find that higher values of ρ_s are associated with lower AE when comparing the same time periods across different ρ_s. Moreover, a higher ρ_s is associated to a lower increase of average AE over time. For example, average AE increased by 18.6% from 2005 to 2022 if ρ_s is 0.75, but 14.6% if ρ_s is 0.88. Blackwood et al. (2021) and Bils et al. (2021) also find that under CRS, less curvature yields lower AE. On average, across the three CRS-related cases (ρ_s = 0.75, 0.85, ρ_s^{DW}), AE increased by approximately 19.4% between 2005 and 2022, and the average AE for the period 2016-2022 was 0.62. This result suggests that there is a 61% potential gain in aggregate productivity in manufacturing that could be achieved by improving resource allocation.

We now turn to the results based on the LP-KG approach. When focusing on cases in which ρ_s is estimated internally from the data (i.e., all cases except ρ_s = 0.75, as a value that is frequently assumed in the literature), we find that average AE tends to be lower once the CRS assumption is relaxed. Moreover, we find that allowing for dispersion in ρ_s across industries has only a modest effect on sectoral AE. This suggests that the variation of demand elasticities among sectors does not substantially alter the overall trend of AE over time. On average, across the two NCRS-related cases (ρ_s = 0.88, ρ_s^{KG}), sectoral AE increased by approximately 21% between 2005 and 2022, and the average sectoral AE for the period 2016-2022 was 0.53. This result implies that improving resource allocation could lead to a potential productivity gain of 89%.

To better understand the mechanisms driving the patterns documented above, we return to equation (9). The second term of this decomposition shows that AE_s depends on the covariance between $TFPR_{is}^{cs}$ and $TFPQ_{is}$. According to Bils et al. (2021), under the core assumptions, log$TFPQ_{is} = \frac{1}{\rho}$ log$TFPR_i^{rr}$.

All else equal, the second term of the decomposition implies that AE_s increases with less dispersion in distortions, $(TFPR_{is}^{cs})$, due to the negative exponent on this term; a weaker positive correlation between distortions and fundamentals, $(TFPQ_{is})$; and given a weaker positive correlation, a lower dispersion in fundamentals, $(TFPQ_{is})$.

Figure 4 shows that the dispersion in $\log TFPQ_{is}$ and $\log TFPR_{is}$, as well as the partial correlation between $\log TFPQ_{is}$ and $\log TFPR_{is}^{cs}$, are quite similar across alternative estimation methods. Overall, we observe a modest reduction in both dispersion and correlation, suggesting an improvement in AE under different specifications.

The analysis in this section highlights several robust conclusions that hold across different elasticity estimates. In all specifications, measured AE has increased over time. Similarly, the dispersion in fundamentals ($\log TFPQ_{is}$) and distortions ($\log TFPR^{cs}_{is}$) has decreased over time, and the correlation between these two variables shows a similar decline across methods and specifications. Together, these patterns imply an increasing trajectory for AE. Furthermore, a reduction in the dispersion of fundamentals and a weaker correlation between fundamentals and distortions work in the same direction to support an increase in AE.

5 Conclusion

This study provides a comprehensive assessment of output and revenue elasticities across industries for the manufacturing sector in Costa Rica, exploring their implications for productivity measurement, firm dynamics, and AE. Several findings emerge from the analysis.

First, the distribution of estimated elasticities reveals systematic differences across methodologies. Output elasticities (CS) tend to be higher for intermediate inputs and lower for capital and labor compared to revenue elasticities (LP, LP-KG). Revenue elasticities remain relatively consistent between the LP and LP-KG methods, closely matching prior estimates in the literature and reinforcing the labor-intensive character of most industries in the sample. The implied revenue curvature and estimated demand elasticities exhibit heterogeneity across sectors, but are centered on values consistent with prior empirical findings.

Second, differences in elasticity estimation methods do not significantly alter broad patterns in productivity dispersion. Measures of within-industry dispersion and correlations across productivity metrics indicate that productivity estimates are highly correlated across methods, and therefore robust for evaluating misallocation. Furthermore, the relationship between firm productivity, growth, and exit is consistent across estimation approaches. Higher productivity is positively associated with firm growth and survival, and larger firms are more resilient to exit, but exhibit slower growth.

Third, our analysis of AE highlights the importance of accounting for sectoral heterogeneity in demand and production structures. Under constant returns to scale (CS), changes in demand elasticity substantially affect AE, with higher demand elasticity leading to lower measured AE. When relaxed the CRS assumption (LP-KG), AE decreases, but shows less sensitivity to variations in ρ_s. Although the magnitude of the AE estimates varies depending on the method, on average, the gains from improving resource allocation are potentially significant. We find that moving to the efficient allocation would increase productivity by 61% - 89% in the manufacturing sector.

In conclusion, the evidence indicates that AE in the manufacturing sector has increased over time; however, its overall level remains relatively low. The modest decline in the dispersion of fundamentals and distortions, coupled with a positive (but weaker) correlation between these factors, points to a continuing upward trajectory for AE.

References

Aguirre, A., Aldunate, R., Arias, A., Azócar, J., Canales, M., Coble, D., Contreras, G., Fernández, A., Fornero, J., Gallardo, I., García, B., Gómez, C., Guerra-Salas, J., Guzmán, D., Huneeus, F., López-Martín, B., Solorza, M., and Taboada, M. (2021). Estimación de parámetros estructurales de la economía chilena. In *Minutas Citadas en Recuadros IPoM Junio 2021*, División Política Monetaria - Banco Central de Chile.

Alfaro-Ureña, A. and Garita-Garita, J. (2018). *Misallocation and Productivity in Costa Rica*, pages 121–148. OECD Economic Survey of Costa Rica: Research Findings on Productivity. OECD Publishing.

Alfaro-Ureña, A., Manelici, I., and Vasquez, J. P. (2022). The effects of joining multinational supply chains: New evidence from firm-to-firm linkages*. *The Quarterly Journal of Economics*, 137(3):1495–1552.

Banco Central de Chile (2017). Productividad total de factores ¿cuánto importa y qué sabemos de sus determinantes? In *Crecimiento tendencial: Proyección de mediano plazo y análisis de sus determinantes.*

Bartelsman, E., Haltiwanger, J., and Scarpetta, S. (2013). Cross-country differences in productivity: The role of allocation and selection. *American Economic Review*, 103(1):305–334.

Bils, M., Klenow, P. J., and Ruane, C. (2021). Misallocation or mismeasurement? *Journal of Monetary Economics*, 124:S39–S56.

Blackwood, G., Foster, L., Grim, C., Haltiwanger, J., and Wolf, Z. (2021). Macro and micro dynamics of productivity: From devilish details to insights. *American Economic Journal: Macroeconomics*, 13(3):142–172.

DeLoecker, J. and Warzynski, F. (2012). Markups and firm-level export status. *American Economic Review*, 102(6):2437–2471.

Eslava, M., Haltiwanger, J., Kugler, A., and Kugler, M. (2013). Trade and market selection: Evidence from manufacturing plants in colombia. *Review of Economic Dynamics*, 16(1):135–158.

Foster, L., Grim, C., and Haltiwanger, J. (2016a). Reallocation in the great recession: Cleansing or not? *Journal of Labor Economics*, 34(S1):S293–S331.

Foster, L., Grim, C., Haltiwanger, J., and Wolf, Z. (2016b). Firm-level dispersion in productivity: Is the devil in the details? *American Economic Review*, 106(5):95–98.

Foster, L., Haltiwanger, J., and Syverson, C. (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review*, 98(1):394–425.

Garcia-Marin, A. and Voigtländer, N. (2019). Exporting and plant-level efficiency gains: It's in the measure. *Journal of Political Economy*, 127(4):1777–1825.

Hall, R. E. and Jones, C. I. (1999). Why do some countries produce so much more output per worker than others? *The Quarterly Journal of Economics*, 114(1):83–116.

Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and manufacturing of TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403–1448.

Hsieh, C.-T. and Klenow, P. J. (2014). The life cycle of plants in india and mexico *. *The Quarterly Journal of Economics*, 129(3):1035–1084.

Ivankovich-Escoto, G. and Martínez-Castillo, J. (2020). La productividad en Costa Rica. Estudios sobre productividad.

Klette, T. J. and Griliches, Z. (1996). The inconsistency of common scale estimators when output prices are unobserved and endogenous. *Journal of Applied Econometrics*, 11(4):343–361.

Levinsohn, J. and Petrin, A. (2003). Estimating production functions using inputs to control for unobservables. *Review of Economic Studies*, 70(2):317–341.

Melitz, M. J. and Polanec, S. (2015). Dynamic olley-pakes productivity decomposition with entry and exit. *The RAND Journal of Economics*, 46(2):362–375.

Monge-González, R. (2019). Productividad y crecimiento económico. Experiencias de algunos países de reciente desarrollo.

Monge-González, R., Crespi, G., and Beverinotti, J. (2020). Confrontando el reto del crecimiento: Productividad e innovación en Costa Rica. Technical report, Inter-American Development Bank.

Olley, G. S. and Pakes, A. (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica*, 64(6):1263.

Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707–720.

Restuccia, D. and Rogerson, R. (2013). Misallocation and productivity. *Review of Economic Dynamics*, 16(1):1–10.

Robles, E. A. (2021). Crecimiento de la productividad total de los factores en costa rica e inestabilidad macroeconómica. *Revista de Ciencias Económicas*, 39(1):1–24.

Syverson, C. (2011). What determines productivity? *Journal of Economic Literature*, 49(2):326–65.

UNDP (2008). International Standard Industrial Classification of All Economic Activities (ISIC), Rev. 4. *Statistical Papers Series M No. 4, Rev. 4*, United Nations, New York.

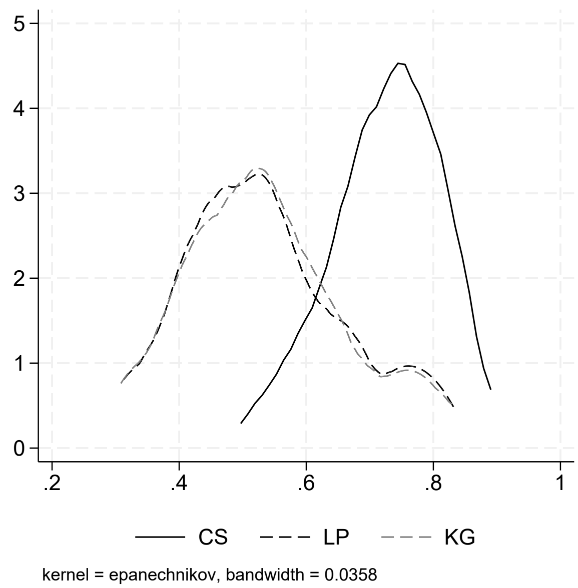
Vega-Monge, M. and Jiménez-Montero, S. (2023). Análisis de productividad en Costa Rica: un enfoque microeconómico.

Vega-Monge, M. and Jiménez-Montero, S. (2025). Análisis de productividad en Costa Rica: un enfoque microeconómico. *RECARD*, 6(1).

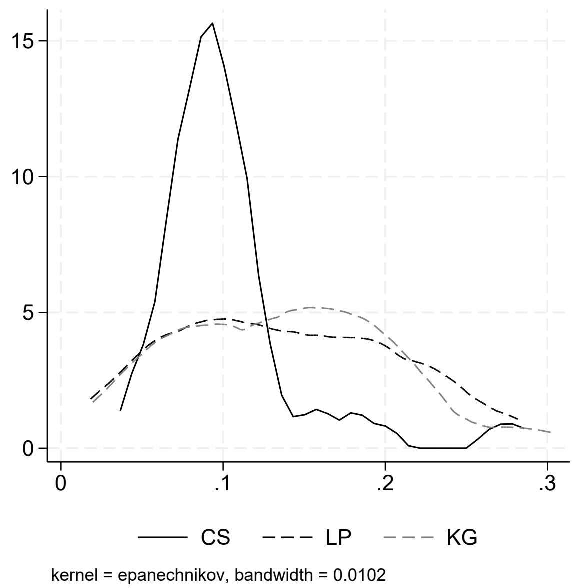
Figures and Tables

Figure 1. Cross-industry distributions of output (α_{js}) and revenue elasticities (β_{js})

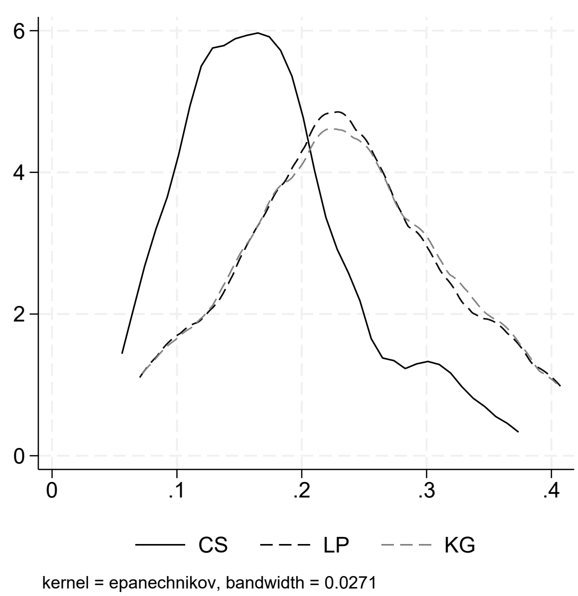
(a) β_{Is} and α_{Is}



(b) β_{Ks} and α_{Ks}



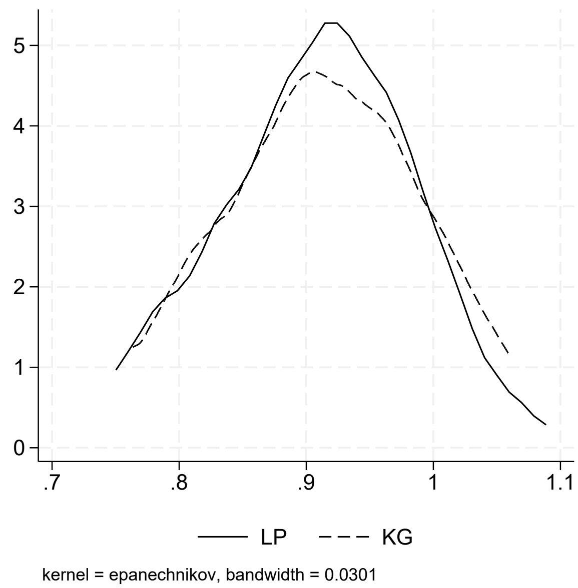
(c) β_{Ls} and α_{Ls}



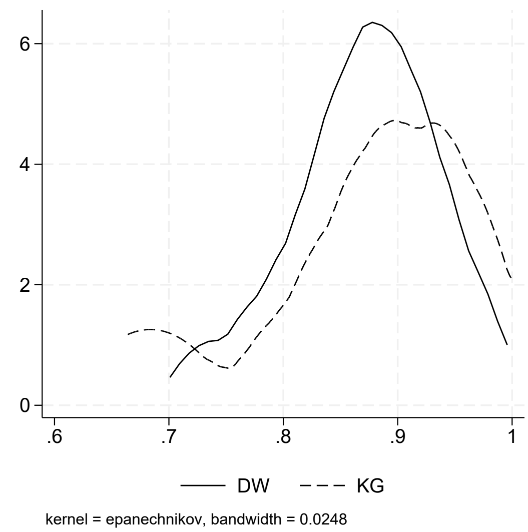
Note. This figure illustrates cross-industry distributions of the estimated output (CS) and revenue elasticities (LP, LP-KG), with respect to intermediate inputs in Panel A, capital in Panel B, and labor in Panel C.

Figure 2. Revenue curvature and its components

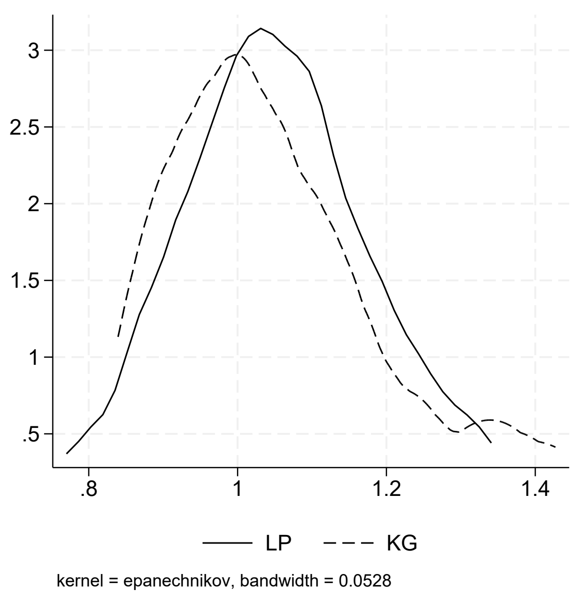
(a) Revenue function curvature: $\widehat{\rho_s \gamma_s} = \sum_j \beta_{js}$



(b) Demand elasticity estimates (ρ_s)

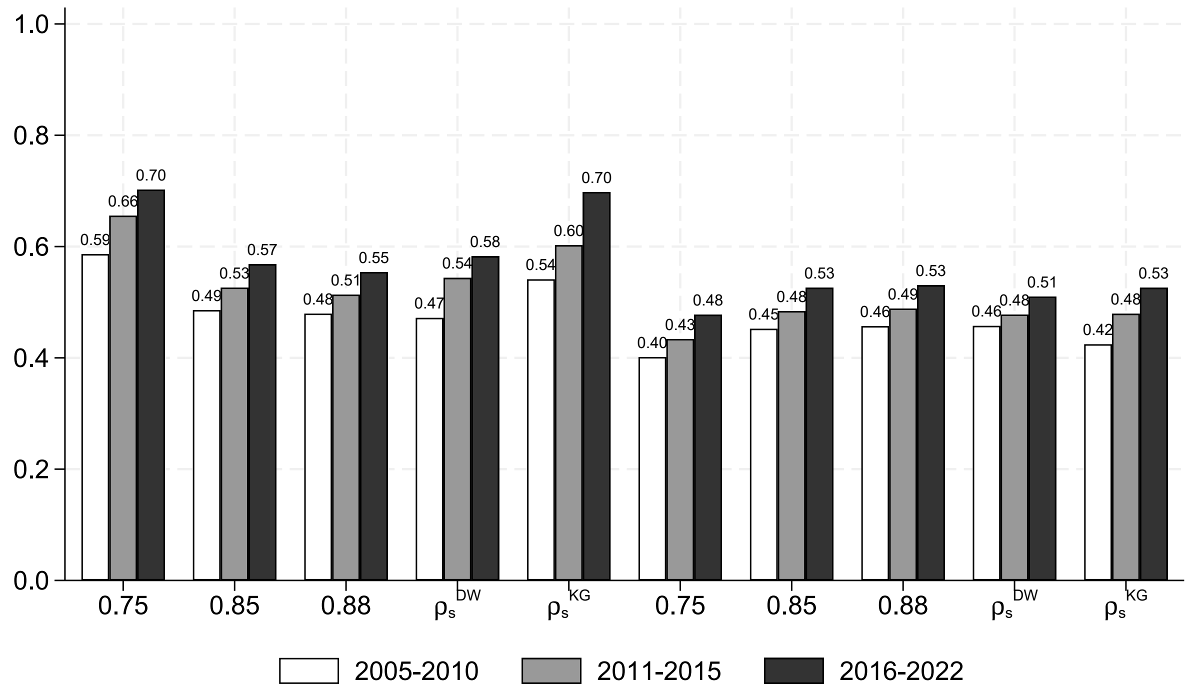


(c) Implied returns to scale (γ_s)



Note. This figure presents the curvature of the revenue function and its components. Panel A displays the the revenue function curvature, Panel B reports the estimated demand elasticities, and Panel C shows the implied returns to scale.

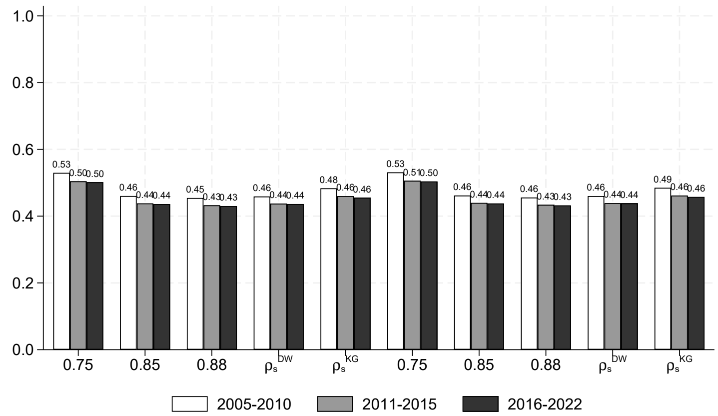
Figure 3. Weighted averaged allocative efficiency, $\sum_s \omega \hat{AE}_s^{COV}$



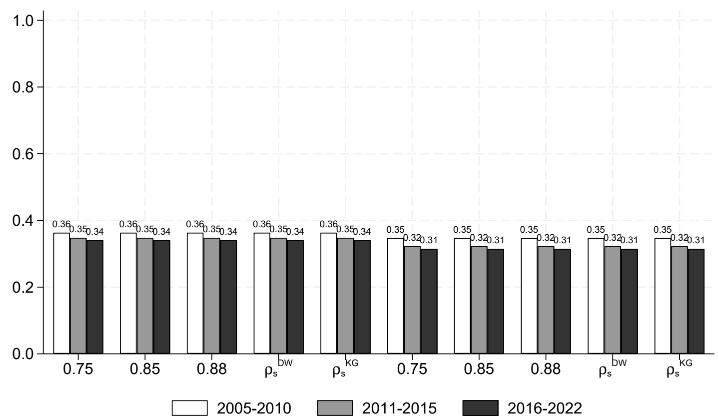
Note: This figure presents weighted averaged allocative efficiency estimates. The x-axis depicts alternative values of ρ_s. ρ_s^{DW} denotes industry-specific time series averages calculated as in De Loecker and Warzynski (2012). ρ_s^{KG} denotes industry-specific time series averages calculated as in Klette and Griliches (1996). The weight ω represents each industry's share of revenue. In industries where $1 < \rho_s \gamma_s$ at the 4-digit level, 1-digit estimates are used.

Figure 4. Productivity moments

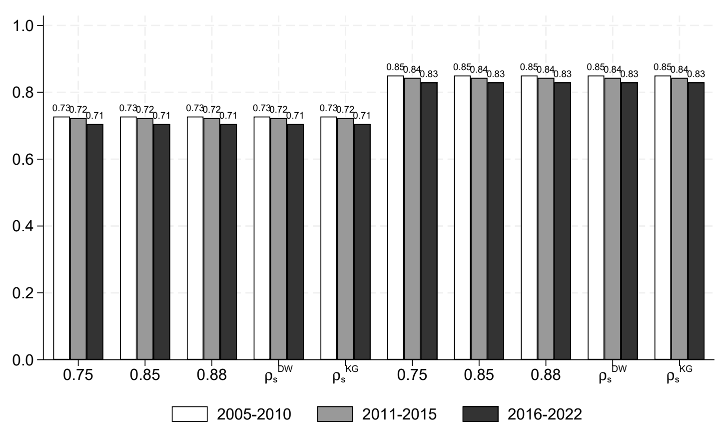
(a) Log$TFPQ$ dispersion



(b) Log$TFPR$ dispersion



(c) Correlation between $\log TFPQ$ and $\log TFPR$



Note: This figure presents productivity moments. Dispersion is measured as a weighted standard deviation, where weights correspond to the number of firm-period observations within each industry. $\log TFPQ$ captures output productivity, while $\log TFPR$ reflects revenue-based productivity.

Table 1. Descriptive statistics of cross-industry elasticities of output and revenue

	CS				LP				LP-KG			
	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
I. Inputs	0.73	0.08	0.53	0.85	0.54	0.13	0.31	0.83	0.54	0.13	0.31	0.83
Capital	0.10	0.04	0.05	0.28	0.14	0.07	0.02	0.28	0.14	0.07	0.02	0.31
Labor	0.17	0.07	0.08	0.35	0.23	0.08	0.07	0.41	0.23	0.08	0.07	0.40

Note. This table shows descriptive statistics of cross-industry elasticities of output (CS) and revenue (LP, LP-KG) with respect to intermediate inputs, capital and labor. The mean is a simple average across industries.

Table 2. Productivity dispersion implied by different models

	Sample size	Interquartile range	Standard deviation
CS	32,430	0.32	0.38
LP	32,430	0.43	0.40
LP-KG	32,430	0.44	0.40

Note. This table presents summary statistics of productivity dispersion across three estimation methods: cost share-based (CS), Levinsohn–Petrin (LP), and Levinsohn–Petrin with demand correction (LP-KG). The interquartile range and standard deviation are computed from weighted distributions, where weights correspond to the number of firm-period observations within each industry.

Table 3. Correlations among within-industry productivity distributions

	A. Pearson			B. Spearman		
	CS	LP	LP-KG	CS	LP	LP-KG
CS	1			1		
LP	0.72	1		0.69	1	
LP-KG	0.72	0.99	1	0.68	0.99	1

Note. This table reports pairwise Pearson (Panel A) and Spearman (Panel B) correlation coefficients between productivity measures computed under different estimation methods: cost share-based (CS), Levinsohn–Petrin (LP), and Levinsohn–Petrin with demand correction (LP-KG).

Table 4. Productivity and size impact on outcomes by productivity estimator

	CS	LP	LP-KG
Panel A. Effect of productivity $(\hat{b}_1)$			
growth	0.05***	0.10***	0.10***
exit	-0.01	-0.02***	-0.02***
conditional growth	0.05***	0.10***	0.10***
Panel B. Effect of log size $(\hat{b}_2)$			
growth	-0.01***	-0.03***	-0.03***
exit	-0.02***	-0.02***	-0.02***
conditional growth	-0.01***	-0.03***	-0.03***

Notes: This table shows $\hat{b}_1$ and $\hat{b}_2$ in equation (11). Outcomes are firm's employment growth (row 1), exit (row 2), and employment growth among continuers (row 3). * $p < 0.1$, ** $p < 0.05$, and *** $p < 0.01$. Robust standard errors in parenthesis.

Online Appendix

A Methodology

A.1 Revenue Productivity Measures

We follow the methodology proposed by Blackwood et al. (2021) that derives a generalized measure of allocative efficiency under constant and non-constant returns to scale. First, the authors employ a production function framework under monopolistic competition (i.e., product differentiation) to estimate the level of sectoral and aggregate productivity. Second, to measure sectoral and aggregate allocative efficiency, they follow two alternative structural approaches that requires decomposing revenue elasticities into returns to scale and markup components (see DeLoecker and Warzynski (2012) and Klette and Griliches (1996)).

The model assumes a Cobb-Douglas production function and a CES demand structure–refered to as "core assumptions" throughout this section, which are common asssumptions in the literature (Hsieh and Klenow, 2009; Bartelsman et al., 2013; Foster et al., 2016b; Bils et al., 2021; Blackwood et al., 2021). Industry (or sector) output is a CES aggregate of intermediate goods producers given by $Q = (\sum_i (\xi_i Q_i)^{\rho})^{\frac{1}{\rho}}$, where $\rho = \frac{\sigma-1}{\sigma}$ with $0 < \rho < 1$ is the demand elasticity, σ is the elasticity of substitution, ξ_i denotes an idiosyncratic demand shifter for plant i, Q_i denotes plant-level quantity, and Q industry level quantity. Time and industry subscripts are omitted in this section in the notation and equations for expositional convenience. The inverse demand function is given by $P_i = PQ^{\frac{1}{\sigma}} Q_i^{-1/\sigma} \xi_i^{\rho}$ for plant i in an industry where P_i denotes plant-level prices and P industry level prices.

The plant-level production function is given by $Q_i = \mathcal{A}_i \prod_j X_{ij}^{\alpha_j}$, where $\mathcal{A}_i$ is technical efficiency, X_{ij} are plant-level factor inputs (e.g., capital, labor, materials, and energy) and α_j is the output elasticity with respect to the jth input X_{ij}. The log of the revenue function is given by:

$$\log P_i + \log Q_i = \sum_j \beta_j \log X_{ij} + \rho \log \mathcal{A}_i + \rho \log \xi_i + (1-\rho) \log Q + \log P \tag{A.1}$$

where the revenue elasticities satisfy $\beta_j = \rho \alpha_j$. Various revenue productivity measures have been used in the theoretical and empirical literature. One typical measure is logTFPR, given by:

$$\log TFPR_i = \log P_i + \log Q_i - \sum_j \alpha_j \log X_{ij} = \log P_i + \log \mathcal{A}_i. \tag{A.2}$$

Equation (A.2) makes explicit that $TFPR_i$ confounds the effect of output prices (P_i) and technical efficiency ($\mathcal{A}_i$). Decomposing $TFPR_i$ into its price and technical efficiency components is generally not feasible. Therefore, the majority of results in the empirical productivity literature are based on revenue productivity measures. An important special case emerges under the assumption that plants minimize total costs and have CRS technology: the share of the jth input expenditure in total costs equal α_{js}. Formally:

$$\log TFPR_i^{cs} = \log P_i + \log Q_i - \sum_j cs_j \log X_{ij} = \log TFPR_i + \sum_j (\alpha_s - cs_j) \log X_{ij} \tag{A.3}$$

where cs_j denotes the cost share of the jth input. Note that $\log TFPR_i$ in equation (A.2) is equivalent to $\log TFPR_i^{cs}$ only under CRS, in which case $\alpha_j = cs_j$. Still, $\log TFPR_i^{cs}$ is of interest in and of itself, even without CRS, since it is indicative of distortions under certain assumptions as Blackwood et al. (2021) demonstrate. This cost-share approach provides a nonparametric estimate of the output elasticities α_j as the jth input expenditures in total costs (even without data on prices and quantities).[16]

The revenue productivity measures above are distinct from the revenue function residual which is given by:

$$\log TFPR_i^{rr} = \log P_i + \log Q_i - \sum_j \beta_j \log X_{ij} = \rho \log \mathcal{A}_i + \rho \log \xi_i + (1-\rho) \log Q + \log P \tag{A.4}$$

which says that the revenue function residual depends on technical efficiency, idiosyncratic demand shocks and aggregate prices and quantities. In addition, the core assumptions allow $\gamma = \sum_j \alpha_j = \rho^{-1} \sum_j \beta_j$, where γ denotes returns to scale and is not necessarily equal to 1.[17] The implications is that $TFPR_i^{rr}$ is different from both $TFPR_i$ and its estimate $TFPR_i^{cs}$.

Without idiosyncratic frictions or distortions, marginal revenue products are equalized across production units and there is no within-industry dispersion in $\log TFPR_i^{cs}$ (which is obviously

[16] For example, $\alpha_j = w_j X_{ij} / \sum_\ell w_\ell X_{i\ell}$, where w_j is the j th factor price.

[17] Importantly, also note that $\sum_j \beta_j = \rho\gamma$, so reducing ρ while keeping $\sum_j \beta_j$ constant requires γ to decrease, for example.

not the case empirically). Since this outcome is counterfactual, Hsieh and Klenow (2009) posit the presence of "distortions" that account for such dispersion. Blackwood et al. (2021) shows that the decision problem of firms who maximize static profits with input distortions implies:[18]

$$TFPR_i^{cs} \propto \tau_i \tag{A.5}$$

where $\tau_i = \prod_j \left(1 + \tau_{ij}\right)^{\alpha_j/\gamma}$ denotes a plant-specific weighted geometric average of input distortions and the weights are given by cost shares. In contrast, Blackwood et al. (2021) shows that $TFPR_i^{rr}$ is proportional to plant technical efficiency and demand shocks under the same assumptions, while abstracting from industry-level shifters that can be captured by industry-year effects:[19]

$$TFPR_i^{rr} \propto \left(\mathcal{A}_i \xi_i\right)^{\rho}. \tag{A.6}$$

One of the key observations from Blackwood et al. (2021) is that $TFPR_i^{cs}$ is proportional to idiosyncratic distortions τ_i while $TFPR_i^{rr}$ is proportional to fundamentals $\mathcal{A}_i \xi_i$. This conceptual difference, due to using output vs. revenue elasticities, is what motivates Blackwood et al. (2021) analysis.

Note that when production exhibits non constant returns to scale (NCRS), $\log TFPR_i^{cs}$ is not equal to $\log TFPR_i$. In this case, $\log TFPR_i^{cs}$ will still only reflect any reduced-form distortions while $\log TFPR_i$ will exhibit dispersion even in the absence of such reduced-form distortions. Furthermore, the finding that $\log TFPR_i^{rr}$ is only a function of fundamentals is robust to deviations from CRS. Consequently, Blackwood et al. (2021) focus on $TFPR_i^{cs}$ and $TFPR_i^{rr}$, since they reflect solely distortions and solely fundamentals, respectively. Hence, they focus on the measures that clearly distinguish distortions and fundamentals, regardless of returns to scale, which are crucial for measuring AE.

A.2 Allocative Efficiency

Recent literature builds on the distinction between $TFPQ$ and $TFPR$ using the core assumptions made in the prior section to construct a measure of misallocation which they term AE.

[18] The profit function in this case is given b y $P_i Q_i - \sum_j w_j (1 + \tau_{ij}^*) X_{ij}$ where w_j denotes the jth input price.

[19] This can be seen from equation (A.4), where industry-year effects capture industry-level shifters P and Q.

The model also collapses the combined effect of demand shifts and technical efficiency that make up $TFPQ_{is}$ for notational convenience: $A_{is} = \mathcal{A}_{is}\xi_{is}$. So the empirical measure of fundamentals is a composite of both demand factors and technical efficiency. Sectoral productivity is defined as sectoral output per composite unit input: $TFPQ_s = Q_s / \prod_j X_{js}^{\alpha_{js}}$. Using CES demand and Cobb-Douglas production with CRS, Bils et al. (2021) show that $TFPQ_s$ can be expressed as a power sum of A_{is} weighted by relative distortions:

$$TFPQ_s = \left(\sum_i A_{is}^{\frac{\rho_s}{1-\rho_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}} \tag{A.7}$$

where $\tilde{\tau}_s$ is the harmonic revenue weighted mean of distortions.[20] Blackwood et al. (2021) show that $TFPQ_s$ is maximized when $\tau_{is} = \tilde{\tau}_s$, whose sufficient condition is satisfied only if $\rho_s < 1$. In this case, following from equation (A.7), $TFPQ_s^*$ is given by $A_s^* = \left(\sum_i A_{is}^{\rho_s/(1-\rho_s)} \right)^{(1-\rho_s)/\rho_s}$. AE is defined as the ratio of $TFPQ_s$ to the maximized, counterfactual $TFPQ_s^*$. Multiplying and dividing by $N_s^{(1-\rho_s)/\rho_s}$, where N_s is the number of plants in the sector, sectoral AE can be expressed as:

$$AE_s = \left(\frac{1}{N_s} \sum_i \left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}} \tag{A.8}$$

where $\tilde{A}_s = \left(N_s^{-1} \sum_i A_{is}^{\rho_s/(1-\rho_s)} \right)$ is the power mean of A_{is}.

Blackwood et al. (2021) also generalize (A.8) to be robust to deviations from CRS but otherwise maintain CES demand and Cobb-Douglas production. Under NCRS, they show that $TFPQ_s$ is given by:

$$TFPQ_s = \frac{\left(\sum_i A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}}}{\left(\prod_j X_{js}^{\alpha_j/\gamma_s} \right)^{1-\gamma_s}} \tag{A.9}$$

They also show that the previous $TFPQ_s$ is maximized when $\tau_{is} = \tilde{\tau}_s$, if $\rho_s\gamma_s < 1$, i.e., firm-specific input distortions are equalized to the harmonic weighted mean of distortions within

[20] $\tilde{\tau}_s = S_1/S_2$ where $S_1 = \sum_i (A_{is}/\tau_{is})^{\rho_s/(1-\rho_s)}$ and $S_2 = \sum_i \left(A_{is}^{\rho_s}/\tau_{is} \right)^{1/(1-\rho_s)}$

sectors. Under these condition, the maximized $TFPQ_s$ is given by:

$$A_s^* = \frac{\left(\sum_i A_{is}^{\rho_s/(1-\rho_s\gamma_s)}\right)^{(1-\rho_s\gamma_s)/\rho_s}}{\left(\prod_j X_{js}^{*\alpha_j/\gamma_s}\right)^{1-\gamma_s}} \tag{A.10}$$

where X_{js}^* denote aggregate input j corresponding to $\max\{TFPQ_s\}$, the case where distortions are equalized across plants. So, AE in this context is the ratio of $TFPQ_s$ (given by equation (A.9)) to the maximized, counterfactual $TFPQ_s^*$ (given by equation (A.10)). Dividing and multiplying by $N_s^{(1-\rho_s\gamma_s)/\rho_s}$, where N_s is the number of plants in the sector, sectoral AE under NCRS can be express as:

$$AE_s = \underbrace{\left(\frac{1}{N_s}\sum_i \left(\frac{A_{is}}{\tilde{A}_s}\right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left(\frac{\tau_{is}}{\bar{\tau}_s}\right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}}\right)^{\frac{1-\rho_s\gamma_s}{\rho_s}}}_{AE_s^{COV}} \underbrace{\left(\frac{\prod_j X_{js}^{*\alpha_j/\gamma_s}}{\prod_j X_{js}^{\alpha_j/\gamma_s}}\right)^{\frac{1-\gamma_s}{\gamma_s}}}_{\text{Sectoral Intermediate Term}} \tag{A.11}$$

where $\tilde{A}_s = \left(N_s^{-1}\sum_i A_{is}^{\rho_s/(1-\rho_s\gamma_s)}\right)^{(1-\rho_s\gamma_s)/\rho_s}$. The first term shows the effect of NCRS on the within-industry component of AE. The second term captures the effect of NCRS via sectoral inputs. Importantly, the latter term equals one when all production factor supplies are exogenous (which is the case we explore in this paper), implying that only AE_s^{COV} is relevant in this case. Also, notice the equivalence in this case between equations (A.8) and (A.11) under CRS $(\gamma_s = 1)$.

The parameters ρ_s, γ_s, and α_{js} affect AE_s^{COV} via the exponents. In addition, ρ_s and γ_s affect relative technical efficiencies and distortions where the denominator depends directly on α_{js}. The implication is that the joint distribution of these variables is a key determinant of AE. Blackwood et al. (2021) formalize this result by expressing AE_s as a function of the covariance between transformations of τ_{is} and A_{is}:

$$\log AE_s^{COV} = \gamma_s \log\left(\frac{\tilde{\tau}_s}{\bar{\tau}_s}\right) + \frac{1-\rho_s\gamma_s}{\rho_s}\log\left[\text{cov}\left(\left(\frac{A_{is}}{\tilde{A}_s}\right)^{\frac{\rho_s}{1-\rho_s\gamma_s}}, \left(\frac{\tau_{is}}{\bar{\tau}_s}\right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}}\right)+1\right] \tag{A.12}$$

where $\bar{\tau}_s = \left(N_s^{-1}\sum_i \tau_{is}^{\rho_s\gamma_s/(\rho_s\gamma_s-1)}\right)^{(\rho_s\gamma_s-1)/\rho_s\gamma_s}$.

Proof Under the assumption that all production factor supplies are exogenous, sectoral AE can be expressed as:

$$AE_s^{COV} = \left(\frac{1}{N_s} \sum_i \left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} \tag{A.13}$$

where $\tilde{A}_s = \left(N_s^{-1} \sum_i A_{is}^{\rho_s/(1-\rho_s\gamma_s)} \right)^{(1-\rho_s\gamma_s)/\rho_s}$.[21] Multiplying and dividing AE_s^{COV} by $\bar{\tau}_s = \left(\sum_i \tau_{is}^{\frac{\rho_s\gamma_s}{\rho_s\gamma_s-1}} \right)^{\frac{\rho_s\gamma_s-1}{\rho_s\gamma_s}}$ and rearranging the resulting expression, we obtain:

$$AE_s^{COV} = \left(\frac{\tilde{\tau}_s}{\bar{\tau}_s} \right)^{\gamma_s} \left(\frac{1}{N_s} \sum_i \left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left(\frac{\tau_{is}}{\bar{\tau}_s} \right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} \tag{A.14}$$

The rest of the demonstration is based on the fact that a sample-based covariance can be written as $\mathrm{cov}(X,Y) = \frac{N}{N-1} \left\{ \frac{1}{N} \sum_i x_i y_i - \bar{x}\bar{y} \right\}$, where $x_i = (A_{is}/\tilde{A}_s)^{\rho_s/(1-\rho_s\gamma_s)}$ and $y_i = (\tau_{is}/\tilde{\tau}_s)^{-\rho_s/(1-\rho_s\gamma_s)}$. Hence,

$$\frac{1}{N} \sum_i x_i y_i = \frac{N-1}{N} \mathrm{cov}(X,Y) + \bar{x}\bar{y}. \tag{A.15}$$

We do not assume that $N/(N-1) = 1$ given that the number of firms within each industry could in principle be small, but the demonstration is more simple when that assumption is made (this is case for Blackwood et al. (2021)). Also, notice that:

$$\bar{x} = \frac{1}{N_s} \sum_i \left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} = 1 \tag{A.16}$$

$$\bar{y} = \frac{1}{N_s} \sum_i \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s}{1-\rho_s\gamma_s}} = 1. \tag{A.17}$$

Substituting accordingly in equation (A.15), we obtain:

$$\frac{1}{N_s} \sum_i \left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}} = \frac{N_s-1}{N_s} \mathrm{cov} \left(\left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s}{1-\rho_s\gamma_s}} \right) + 1. \tag{A.18}$$

Hence

$$\log AE_s^{COV} = \gamma_s \log \left(\frac{\tilde{\tau}_s}{\bar{\tau}_s} \right) + \frac{1-\rho_s\gamma_s}{\rho_s} \log \left(\frac{N_s-1}{N_s} \mathrm{cov} \left(\left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s}{1-\rho_s\gamma_s}} \right) + 1 \right) \tag{A.19}$$

[21] Under CRS ($\gamma_s = 1$), $AE_s^{COV} = \left(\frac{1}{N_s} \sum_i \left(\frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s}} \left(\frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{-\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}}$.

Under the assumption that $(N_s - 1)/N_s = 1$, equation (A.19) is equivalent to equation (A.12).

Equation (A.12) shows that AE_s^{COV} depends on sectoral distortions (term 1), and a function of the covariance between exponentiated relative technical efficiencies and distortions (term 2). By definition, the covariance (term 2) depends on the dispersion of these two variables and the correlation between technical efficiencies and distortions.

We use equation (A.12) in our empirical analysis below to provide guidance about the sensitivity of the measures of AE to the estimates of ρ_s, γ_s, and α_{js}. Studying such sensitivity is important because of the complex relationship between curvature parameters and AE. The second term of equation (A.12) highlights that ρ_s and γ_s do not enter AE symmetrically, so the influence of the two parameters cannot be summarized by revenue curvature. Further complicating matters, the empirically measured distributions of fundamentals, including the correlation between fundamentals and distortions, depends asymmetrically on ρ_s and γ_s. This in turn implies the covariance of measured $TFPQ_{is}$ with measured $TFPR_{is}^{cs}$ varies with ρ_s and γ_s in an asymmetric manner. This highlights the importance of using different estimates for ρ_s and γ_s in the empirical analysis.

Critically to highlight the connection between $\log AE_s^{COV}$, $\log TFPR_{is}^{cs}$, and $\log TFPR_{is}^{rr}$ is that $\tau_{is} \propto TFPR_{is}^{cs}$ and $\log A_{is} = \frac{1}{\rho_s} \log TFPR_{is}^{rr}$. Hence, the empirical equivalent of equation (A.12) can be re-written as:

$$\log AE_s^{COV} = \gamma_s \log \frac{\widetilde{TFPR}_s^{cs}}{\overline{TFPR}_s^{cs}} + \frac{1-\rho_s\gamma_s}{\rho_s} \log \left[\operatorname{cov} \left(\left(\frac{TFPR_{is}^{rr}}{\widetilde{TFPR}_s^{rr}} \right)^{\frac{1}{1-\rho_s\gamma_s}}, \left(\frac{TFPR_{is}^{cs}}{\overline{TFPR}_s^{cs}} \right)^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right) + 1 \right] \tag{A.20}$$

In other words, the measured AE_s^{COV} is a function of the two distinct revenue productivity measures derived above. Our empirical analysis is grounded in equation (A.20).