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Carlos Segura Rodríguez

José Pablo Vásquez Carvajal

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Carlos Segura Rodríguez[*], José Pablo Vásquez Carvajal[†]



## Abstract

Nowadays, it is very common to use the method proposed by Hodrick and Prescott (1980) to split a time series in a trend and a cyclical component. Its use concentrates primarily on the fluctuation analysis of the economic cycles, which were defined by Lucas (1977) as deviations of the real product from a trend. In this context, the HP filter is very useful for the estimation of the potential product as the trend component of the observed product. This paper aims at amplifying the information used in the previous work by Esquivel and Rojas (2007), analyzing an alternative methodology proposed by Marcet and Ravn (2003) and comparing its results with the formerly used methodology.

**Key words:** Hodrick-Prescott filter.

**JEL codes**: C49, C65.

[*] Department of Economic Research. Email address segurarc@bccr.fi.cr

[†] Department of Economic Research. Email address vasquezcj@bccr.fi.cr

# Estimación del parámetro de suavizamiento del filtro de Hodrick y Prescott para Costa Rica

Carlos Segura Rodríguez‡, José Pablo Vásquez Carvajal §



## Resumen

En la actualidad, es muy común utilizar el método propuesto por Hodrick y Prescott (1980) (HP filter) para descomponer una serie de tiempo en una tendencia y un componente cíclico. Su uso se concentra principalmente en el análisis de las fluctuaciones de los ciclos económicos, los cuales fueron definidos por Lucas (1977) como desviaciones del producto real con respecto a una tendencia. En este sentido, el filtro HP resulta de mucha utilidad para la estimación del producto potencial como la parte tendencial del producto observado. El presente estudio pretende ampliar la cantidad de información utilizada en el trabajo previamente realizado por Esquivel y Rojas (2007), analizar una metodología alternativa propuesta por Marcet y Ravn (2003) y contrastar los resultados con la metodología utilizada anteriormente.

**Palabras clave:** Filtro Hodrick-Prescott.

**Clasificación JEL**: C49, C65.

‡ Departamento de Investigación Económica. División Económica, BCCR. segurarc@bccr.fi.cr

§ Departamento de Investigación Económica. División Económica, BCCR. vasquezcj@bccr.fi.cr

# Estimation of the Hodrick and Prescott filter smoothing parameter for Costa Rica

*Carlos Segura Rodríguez*[1]
*José Pablo Vásquez Carvajal*[2]

## 1. Introduction

Nowadays, it is very common to use the method proposed by Hodrick and Prescott (1980) (HP filter) to split a time series in a trend and a cyclical component. Its use concentrates primarily on the fluctuation analysis of the economic cycles, which were defined by Lucas (1977) as deviations of the real product from a trend. In this context, the HP filter is very useful for the estimation of the potential product as the trend component of the observed product.

The HP filter minimizes the sum of the squares of the trend deviations penalizing the changes in the series trend acceleration. The penalty or smoothing parameter is known as lambda (λ) and has to be chosen by the researcher before the beginning of the procedure; this constitutes one of the main aspects that have been criticized about it.

The main problem of this method lies in the arbitrariness with which the smoothing parameter is chosen. The choice made by Hodrick and Prescott on this parameter for quarterly data was based in an *a priori* consideration about the variability of the cyclical and trend component of the American economy.

> "If the cyclical components and the second differences of the growth components were identically and independently distributed, normal variables with means zero and variances $\sigma^2_1$ and $\sigma^2_2$, (which they are not), the conditional expectation of the $y_t^{tr}$, given the observations would be the solution to the program if $\sqrt{\lambda} = \frac{1}{}$ […].Our prior view is that five percent cyclical component is moderately large as is a one-eighth of one percent change in the growth rate in a quarter. This led us to select $\sqrt{\lambda}$ – or 1600" (Hodrick and Prescott, 1980).

However, despite the fact that the choice of these authors produces reasonable results for American data, there is no guarantee that the same value for λ will provide similar results for other countries, because the properties of the economic cycles can differ among them.

There is a previous study by Esquivel and Rojas (2007) where production data from 1991 to 2006 is used to estimate the most appropriate values of λ for Costa Rica, following a methodology proposed by Marcet and Ravn (2003). This methodology tries to make the smoothing parameters, calculated for a

[1] segurarc@bccr.fi.cr
[2] vasquezcj@bccr.fi.cr

specific country, comparable with the respective values of 14400, 1600, and 100, used in a standard way in the American case for monthly, quarterly and yearly data, respectively.

This paper aims at amplifying the information used in the previous work by Esquivel and Rojas (2007), analyzing an alternative methodology proposed by Marcet and Ravn (2003) and comparing its results with the formerly used methodology.

The document is structured as follows. In section 2 the original method of the HP filter is analyzed briefly, including some of the criticism against it. Section 3 outlines the methodology to estimate the smoothing parameters for Costa Rica, and the data used to calculate this estimation. In section 4 the criteria for the selection of the optimum value of λ and the main results are presented. Section 5 concludes.

## 2. The Hodrick and Prescott filter

To calculate the smoothing parameter of the American Gross Domestic Product (GDP), Hodrick and Prescott (1980) assume that any time series can be expressed as follows:

$$y_t = y_t^{tr} + y_t^c$$

where $y_t$ represents the observed series, $y_t^{tr}$ represents the non-observable trend component at time t and $y_t^c$ represents the cyclical residual component at time t. The trend calculated by the HP filter is obtained as the solution the following optimization problem:

$$\min_{\{y_t^{tr}\}_{t=1}^{T}} \left[\sum_{t=1}^{T}(y_t - y_t^{tr})^2 + \lambda \sum_{t=2}^{T-1}\left((y_{t+1}^{tr} - y_t^{tr}) - (y_t^{tr} - y_{t-1}^{tr})\right)^2\right] \quad with\ \lambda > 0$$

The first term is a measure of the trend adjustment to the original series. The second term seeks to minimize the changes in the curvature of the trend, represented in the formula by the second differences of the trend as an approximation of the second derivative of $y_t$. The relative importance between the trend adjustment and the variability of its acceleration (changes in the curvature) is weighted by the parameter λ. As the value of λ grows, the trend estimated by the HP filter becomes smoother. In the extreme case, when λ tends to infinity, the estimated trend becomes linear; allowing big fluctuations of the cycle. On the other hand, when λ tends to zero, the trend component becomes equivalent to the original series $y_t$, which reduces the cyclical component to zero.

There are two important aspects that have to be taken into account. First, the HP filter has to be applied to seasonally adjusted series so that the cyclical component will not be contaminated by seasonal variations. Second, the Hodrick and Prescott method (and all of its variations) has the inconvenience of estimating a trend series which is very sensitive to transitory shocks at the end of the sample; this is known as the tail problem (or end of sample problem). To reduce this inconvenient fact, it is recommended to calculate forecasts of one or two years from the original series before calculating the trend of a series with the HP filter. (Maravall and Kaiser, 2002)

Despite the popularity of the Hodrick and Prescott method, it has been subject of a fair amount of criticism. For example, King and Rebelo (1993) argue that the HP filter is ideal in the sense of minimizing the sum of squared errors only for a limited kind of ARIMA models that are not close to what happens in practice. Other authors (see Harvey and Jaeger (1993), Cogley and Nason (1995)) criticize the possibility of obtaining spurious cycles when applying the filter. On the other hand, Maravall and del Río (2001) state that the HP filter does not preserve trends estimated under temporal aggregation or disaggregation of the series using the standard values of λ; which means that applying the HP filter to aggregated series is not equivalent to applying the filter to disaggregated series and then aggregating them. Also, Marcet and Ravn (2003) question the fact that one of the conditions for the HP filter to be optimal is that the cyclical component show no autocorrelation, which is not true in general. According to these authors, when the cyclical component does show autocorrelation, the estimation of the HP filter underestimates the variance of the component.

The most relevant criticism for this case is that of Marcet and Ravn (2003). Their argument entails that if λ is maintained constant for all countries, a greater part of the cyclical component would be assigned to the trend in countries with a higher autocorrelation in their cycle, which would result in an estimation of a more volatile trend than the real one.

For this reason it is recommended to use smoothing parameters that are consistent with the economic cycle of every country. Otherwise, it would be implicitly supposed that each country has an economic cycle with identical characteristics to that of the U.S.

## 3. Methodology

Marcet and Ravn (2003) develop two procedures that allow the comparison of the variability of trends between different countries, starting from the value proposed by Hodrick and Prescott for λ in the case of the U.S. Both methods minimize the sum of squares of the trend deviations from the original series, but differ in the applied restrictions. These approaches try to endogenously obtain a value for λ that is consistent with the imposed restrictions, according to the characteristics of each country. Both methodologies will be explained below.

**Methodology with relative variability of the trend acceleration**

It consists in the minimization of the sum of squares of the difference between the original series and the trend component estimated for a specific country, subject to the fact that the variability of the acceleration in the relative trend to the variability of the cyclical component is limited from above by a positive constant V:

$$\min_{\{y_t^{tr}\}_{t=1}^{T}} \left[ \sum_{t=1}^{T} (y_t - y_t^{tr})^2 \right]$$

$$s.t.: \frac{\sum_{t=2}^{T-1} ((y_{t+1}^{tr} - y_t^{tr}) - (y_t^{tr} - y_{t-1}^{tr}))^2}{\sum_{t=1}^{T} (y_t - y_t^{tr})^2} \leq V$$

The authors demonstrate that for the appropriate values of λ and V the former problem and the HP filter are equivalent. If V=0, the described problem results in a linear trend, whereas when V tends to infinity the trend is identical to the original series. This means that this methodology shows the same flexibility as the one originally proposed by Hodrick and Prescott.

According to Marcet and Ravn (2003), maintaining the same value for V for all the countries assures comparability between them, in the sense that the variability of the acceleration of the trend relative to the variability of the cyclical component is the same.

For practical effects, V is considered the value obtained when evaluating the above inequality with the obtained trend results in the case of the U.S., using the Hodrick and Prescott filter with the standard values for λ. Once this is done, the value of the smoothing parameter is calculated for the studied series by iterating until the restriction of this series and the one calculated for the U.S. are equal. That is, given the value of λ for the U.S., a value of λ for Costa Rica has to be found that equals the values of V for both countries. From now on, this procedure will be called methodology with V.

**Methodology with the variability of the trend's acceleration**

This methodology considers the same objective function as the minimization problem above, but with a different restriction given by:

$$\frac{\sum_{t=2}^{T-1}((y_{t+1}^{tr}-y_t^{tr})-(y_t^{tr}-y_{t-1}^{tr}))^2}{T-2} \leq W$$

where W is the upper limit of the trend component's variability of acceleration. When estimating λ, the same procedure as in the above methodology has to be applied. This process will be called methodology with W.

Marcet and Ravn (2003) state that in practice it is sufficient to adjust the value of λ upwards when the difference between the value of V (W) for a specific country and the value of V (W) for the U.S. is positive and vice versa; due to the fact that they did not find multiplicity of solutions for the first order conditions in their study[3].

Regarding data, values for λ were estimated using monthly, quarterly and annual production series of Costa Rica and of the U.S. from 1983 to 2010. For the case of the monthly series, data from the Monthly Economic Activity Index IMAE(for its Spanish acronym) for Costa Rica and from the Industrial Production Index (IPI) for the U.S. was used, as well as from the monthly gross domestic product (GDP) for both countries. This one was estimated on the basis of the quarterly data of the GDP using the ECOTRIM program, taking the IMAE as a reference series for the case of Costa Rica and the IPI for the case of the U.S.

[3] For the case of Costa Rica, we found no multiplicity of solutions during the process of iteration in the case of the quarterly series, even when the value of λ was adjusted upwards as well as downwards regardless of the sign of the difference between the value of V (W) for Costa Rica and the value of V (W) for the U.S.

The following chart summarizes the used data series:

| Table 1. Used data series | |
|---|---|
| **Frequency** | **Series** |
| **Quarterly** | Seasonally adjusted GDP |
| **Yearly** | Yearly GDP |
| **Monthly** | 1. IMAE for the case of Costa Rica and IPI for the case of the U.S., both seasonally adjusted.<br>2. Estimated seasonally adjusted monthly GDP |

#### 4. Results

Before proceeding to the estimation process with the methodologies proposed by Marcet and Ravn (2003), it is corroborated that there is no evidence of autocorrelation in the cycle in the case of Costa Rica when it is estimated with the standard values of the smoothing parameter; thus, the estimations of the trend with these parameters are not the optimal estimations for the Costa Rican case. In Table 2, the autocorrelations of the estimated cycles are shown for each one of the series used in the study.

As it can be seen in the chart, the results prove the presence of a correlation significantly different from zero, which is consistent with the objection stated by Marcet and Ravn (2003); it confirms that the estimations obtained with the conventional values of the HP filter impair the comparability of the trends of the studied series for the case of Costa Rica with those of the U.S.

**Table 2. Autocorrelations of the estimated cycles with standard values of $\lambda$**

| | IMAE | Monthly GDP | Quarterly GDP | Yearly GDP |
|---|---|---|---|---|
| **Autocorrelation** | 0.467 | 0.520 | 0.802 | 0.481 |
| **Barlett Standard Deviation** | 0.055 | 0.055 | 0.094 | 0.189 |
| **T Student** | 8.491 | 9.455 | 8.532 | 2.545 |

#### 4.1 Choice of the optimal value for $\lambda$

Due to the fact that the procedures proposed by Marcet and Ravn (2003) yield to two values of $\lambda$ that minimize the square deviations of the estimated trend of the original series, they recommend using the methodology with W if the researcher believes that the deviation of the true trend regarding a linear trend is similar among the countries. For example, when two countries share mutual industrial structures or are exposed to similar economic conditions. On the other hand, using the methodology with V is recommended if the researcher thinks that the deviations of the linear trend are greater in one of the countries considered. Nevertheless, this consideration can imply the fact that the researchers may choose one of the methodologies without considering objective criteria during the decision process.

Therefore, it is necessary to have additional indicators that allow a better selection between both methods. In this study, the decision was made based on the existing consistency between the trends of the estimated series in different frequencies that are obtained by using the different considered smoothing parameters, given that the lack of consistency in this aspect has been one of the most severe criticisms for the use of the HP filter (for reference, see Maravall and del Río (2001), Central American Monetary Council (2004)).

Maravall and del Río (2001) suggest to take the estimations made on the basis of the quarterly series as a point of reference to evaluate the consistency of the component estimations for the same series in different frequencies, because there is a consensus among researchers from outside the U.S. about the use of $\lambda$=1600 for quarterly series than about the values of $\lambda$ that should be used for data in other frequencies. Hence, from now on the comparisons made will take the estimated values of $\lambda$ for the Costa Rican quarterly series as a reference.

For the development of their criticism against the HP filter, they consider two series of the same variable with different frequencies: one reference series and one compared series, to which the HP filter is applied with $\lambda_1$ and $\lambda_2$ as values for the smoothing parameter, respectively.

The authors state that the process that should be carried out to find values of $\lambda$ consistent under aggregation of series is to consider the $\lambda_1$ value fixed and then to find a value $\lambda_2$ for the other series such that the components obtained with direct and indirect estimation are as close as possible; where the direct estimation refers to the one estimated applying $\lambda_1$ to the reference series, and the indirect estimation is the result obtained by aggregating the estimation of the HO filter for the second series with the value of $\lambda_2$. The case of a comparison regarding a disaggregated series is considered in a similar way.

Thus, the adjustment of the aggregation of the trend for the monthly GDP series to the yearly series estimated with respect to the aggregated trend based on the quarterly GDP series can be considered, which means that the first two series are compared with the last one after aggregating data until obtaining yearly periodicities. This comparison can be carried out by using goodness of fit measures like the Mean Squared Error (MSE) or the Mean Absolute Deviation (MAD). In Table 3, the results for the Costa Rican GDP series are shown.

As it can be seen in the chart, the precision of the estimated trend is higher when the value of $\lambda$ obtained by using the procedure determined by W is applied, for the monthly series as for the yearly series.

| Table 3. Goodness of fit for the aggregation of monthly and yearly estimations regarding quarterly estimations with HP filter | | | |
|---|---|---|---|
| | | Minimization restricted by V | Minimization restricted by W |
| **Monthly GDP** | MSE | 201.91 | 85.19 |
| | MSE | 11.14 | 6.32 |
| **Yearly GDP** | MAD | 149.82 | 63.84 |
| | MAD | 7.87 | 4.94 |

On the other hand, the Central American Monetary Council (2004) recommends applying the value for $\lambda$ that minimizes the difference in the range of variation of the growth rates of the estimated trend for each one of the frequencies of the considered variable. This is based on the same principle of favoring the values of $\lambda$ that show the most consistent results[4].

The differences in the range of growth rates occur because more variations throughout time can be detected if high frequency series are considered because these variations are diluted when using data with a higher aggregation. This visibility of events in economy with high frequency series results in higher levels of variability in these cases; nevertheless, the biggest part of this variability should not affect the estimated trend.

In Table 4 the results of the current research are shown, considering the quarterly estimation as the object of comparison. The table shows that the differences (in absolute terms) between the ranges of the growth rates of the trend regarding the ranges in the case of the quarterly estimations are smaller for all the considered periodicities when using the W procedure.

| Table 4. Comparison of the range of the growth rates of the estimated trends regarding the quarterly data | | | | |
|---|---|---|---|---|
| **SERIES** | V | | W | |
| | Growth rate range | Quarterly differences | Growth rate range | Quarterly differences |
| **IMAE** | 8.06 pp | 4.52 pp | 4.64 pp | 2.07 pp |
| **Monthly GDP** | 6.32 pp | 2.78 pp | 3.85 pp | 1.28 pp |
| **Quarterly GDP** | 3.54 pp | --- | 2.57 pp | --- |
| **Yearly GDP** | 1.15 pp | 2.39 pp | 1.02 pp | 1.55 pp |

[4] The theoretical principle is also based on the consideration of the work presented by Maravall and del Río (2001).

Finally, Maravall and del Río (2001) demonstrate that considering the aggregation of series from a fixed point, the distribution of probability that the cycles with different periodicity can be excluded from the trend estimation can also be calculated. The estimator of the cycle's frequency that has a 50% probability of being excluded from the trend is given by the following formula[5]:

$$\tau = {}^{2\pi}\Big/_{\cos^{-1}\left(1-\frac{1}{2\sqrt{\lambda}}\right)}$$

where τ is the estimation of the frequency and λ corresponds to the value of the smoothing parameter used in the estimation of the Hodrick and Prescott filter.

The value of this parameter (τ) can be used to compare the consistency in the frequency of the cycles excluded from the trend with series measured in different periodicity. Nonetheless, trying to equal the value of this parameter can lead to extreme values for the application of the filter. These values were estimated by Maravall and del Río (2001) as λ=6.65 for yearly series and λ= 129119 for monthly series comparing them to the values of a quarterly parameter of 1600. Hence, estimations for the value τ have to be found, that are as similar as possible, but not necessarily equal.

In Table 5 the obtained results for the case of the Costa Rican production series are shown. The differences in absolute terms, regarding the frequency of cycles estimated for the quarterly series can be observed to be similar in the case of the IMAE, while the W methodology shows better performances in the case of the monthly and the annual product.

**Table 5. Difference in the frequency of the cycles (yearly)**

| SERIE | V | | W | |
|---|---|---|---|---|
| | Years | Difference | Years | Difference |
| **IMAE** | 4.0342 | 4.3898 | 6.4549 | 4.3708 |
| **Monthly GDP** | 4.7813 | 3.6428 | 7.5646 | 3.2612 |
| **Quarterly GDP** | 8.4241 | | 10.8257 | |
| **Yearly GDP** | 13.9308 | 5.5068 | 15.0625 | 4.2367 |

Based on the before presented data, it can be concluded that the best estimation of the HP filter in the case of Costa Rica is obtained by using the values for λ calculated with the W methodology; therefore it is recommended to keep on using these values whenever it is necessary to calculate the trend and the cycle of the series.

[5] The estimation has a probability distribution associated, so that cycles of different periodicity can be excluded from the trend. For example, for λ=1600 in the case of a quarterly series, the frequency in which the probability of exclusion is 50% is 10 years, and it can be stated with statistical certainty that, approximately, cycles shorter than 6 years will be excluded from the trend, while those that are longer than 16 years will be included in it. (Maravall and del Río, 2001)

Below, the main results of this methodology are presented. As a reference, the results obtained with the V methodology are contained in the appendix.

#### 4.2 Results with W.

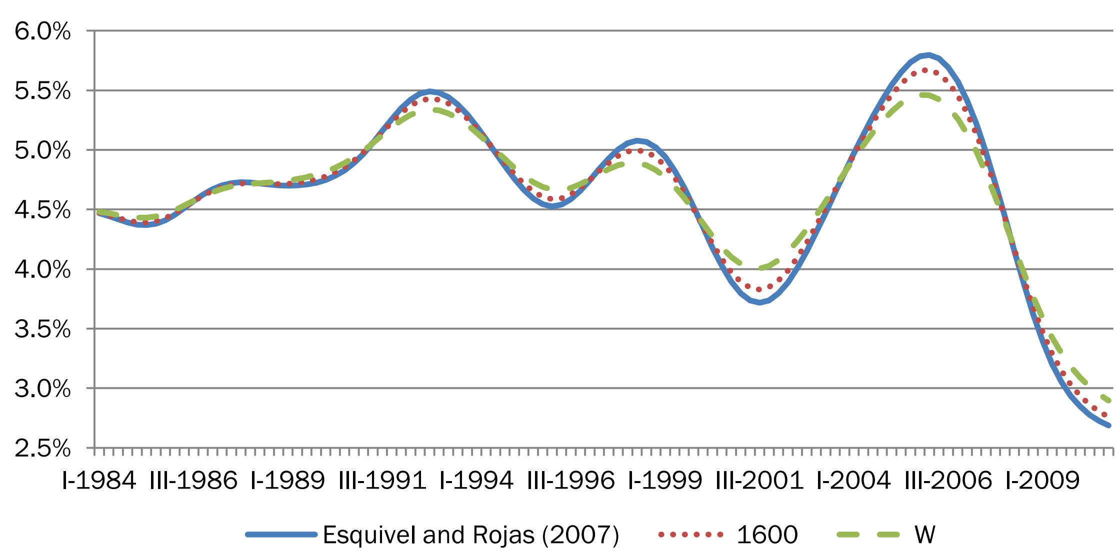
This section presents a description of the realized process and the intermediate results for the variable in quarterly periodicity. Then, the final results for the rest of the estimations are described.

The first step is estimating the values of W for each of the countries after applying the filter with a value of $\lambda$=1600, which yields $W_{EEUU}$=6.02*10-8 and $W_{CR}$=8.95*10-8 for the quarterly case. It shows that in this estimation, the variability of trend acceleration for Costa Rica is allowed to be nearly 1.5 times the value that is obtained for the case of the U.S.

Due to the fact that the result for Costa Rica is higher than that for the U.S., an iterative process is carried out in order to increase the value of the parameter $\lambda$ in unit steps, as described in the methodology, until the calculated value for $W_{CR}$ equals the estimated value for the case of the USA. This procedure shows that the optimum value of the smoothing parameter for the estimation of the Hodrick-Prescott filter in the case of Costa Rica is $\lambda$=2250.

Graph 1 contains a comparison of the year-on-year variation rates of the product trend estimated with the Hodrick-Prescott filter using three different values of $\lambda$: the standard value, the one calculated as optimal by Rojas and Esquivel (2007), and the estimation obtained in this study; 1600, 1311, and 2250, respectively.

Graph1. Year-on-year potential product variation rates
Quaterly series. Costa Rica 1984-2010



It can be observed that the graph is consistent with the idea that of the optimal estimation result in rates of the trend acceleration that are lower than those that could have been estimated by using the filter with the standard value of the parameter[6].

Table 6 summarizes the results obtained when repeating this process for each one of the series considered in this study, the values obtained by Esquivel and Rojas (2007), and the standard values for the parameter λ.

It can be concluded that the highest divergences regarding the estimated λ occur in the monthly and quarterly series and not in the yearly one, at least compared to the estimated values of the prior study.

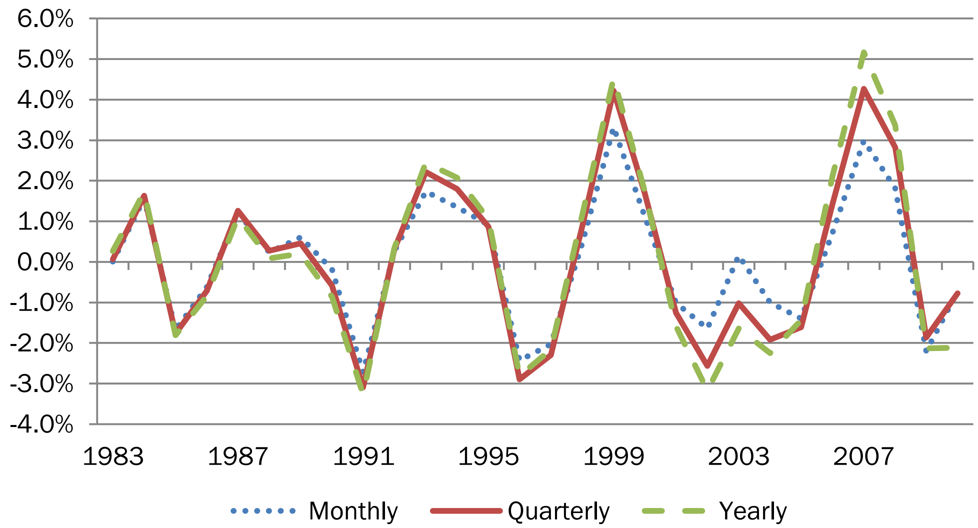
**Table 6. Estimations of the optima values for λ**

| SERIES | New | Esquivel and Rojas (2007) | Standard Values |
|---|---|---|---|
| **IMAE** | 23000 | 14087 | 14400 |
| **Monthly GDP** | 44000 | 14087 | 14400 |
| **Quarterly GDP** | 2250 | 1311 | 1600 |
| **Yearly GDP** | 35 | 41 | 100 |

The following graph shows the product gap calculated as the difference between the observed yearly series and the trend estimations defined by using the HP filter for each one of the product series of different frequencies, which were aggregated until obtaining yearly periodicity series, weighted by the same trend.

[6] However, the values for the average growth rate of the estimated trend do not vary with the usage of different values for λ and it is estimated as 4.64% for the case of the series in quarterly frequency (4.70% in the case of the yearly series).

Graph 2. Costa Rica: Output Gap
1983-2010 (as a percentage of the potential GDP)




This graph also shows that the results for the product gap obtained with the three series are very consistent. Only the product gap estimated with the yearly series show a dissimilar behavior at the end of the sample that may exist due to a residual of the tail problem caused by the estimation of the filter.

## 5. Conclusions

This paper shows that using standard values of λ for the Costa Rican case leads to estimating trends with a different variability in their acceleration, from that obtained for the case of the U.S. According to Marcet and Ravn (2003), searching for the values of that λ which make the trend estimations comparable for both countries is recommendable.

For the Costa Rican case, the W methodology proposed by these authors resulted to be more consistent considering the aggregation or disaggregation process of one series in different frequencies. For this reason it is estimated that values for λ of 44000, 2250 and 35 are the optimal values for monthly, quarterly and yearly GDP data in the case of Costa Rican economy; and 23000 for the case of the IMAE. Nevertheless, due to the fact that there are two different values for the smoothing parameter in the case of monthly series it is recommended to use the one related to the IMAE, because it is the indicator used as a reference for approximating the monthly GDP.

From now on, the values recommended of the parameter $\lambda$ for the Costa Rican economy are:

**Table 7. Optimal smoothing parameters**

| Periodicity | Optimal λ |
|---|---|
| **Yearly** | 35 |
| **Quarterly** | 2250 |
| **Monthly** | 23000 |

These results are a contribution to the initial proposal of Esquivel and Rojas (2007), as a wider period was covered and a new methodology was applied to calculate $\lambda$ which proved to be the more appropriate.

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## 7. Appendix

### Appendix A. Results with the V methodology

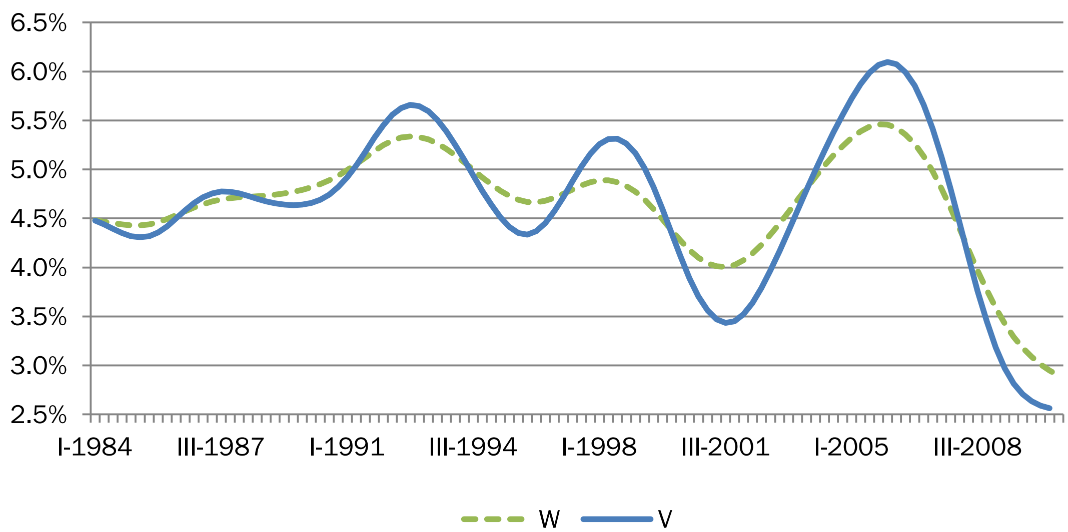
This appendix contains the main results of estimating the parameter with the V methodology. First, it can be observed in the following chart that generally, the optima values of $\lambda$ are very far below those estimated with the W methodology. (See Table 6).

**Table 8. Estimations of optima $\lambda$ values with V methodology**

| SERIE | New | Esquivel y Rojas (2007) | Standard values |
|---|---|---|---|
| **IMAE** | 3500 | 14087 | 14400 |
| **Monthly GDP** | 7000 | 14087 | 14400 |
| **Quarterly GDP** | 830 | 1311 | 1600 |
| **Yearly GDP** | 25 | 41 | 100 |

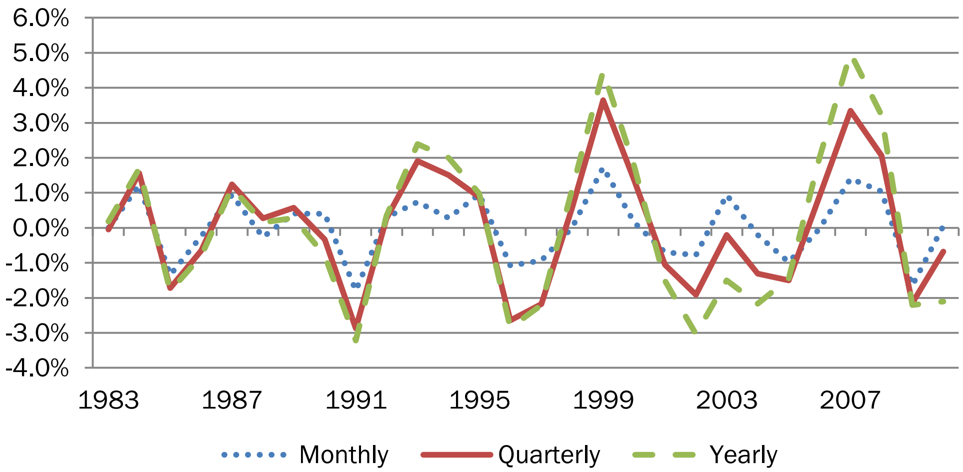
These results entail, as it can be seen in Graph 3, that the estimations of the variability trend growth rates are higher than those estimated with the W methodology for the case of the series in quarterly frequency.

Graph 3
Year-on-year variation rates of the quarterly potential product.
Costa Rica 1984-2010




Finally, the estimation of the product gap estimation after applying the HP filter to each one of the series in different frequencies using the optimal values according to the V methodology is presented. As it can be seen, the variations between each of the estimated cycles differ stronger than those calculated with the W methodology.

Graph 4. Costa Rica: Output Gap
1983-2010 (as percentage of the potential GDP)




**Appendix B. Obtained results for the period 1991-2010**

The same procedure was applied to a part of the considered sample (from 1991 on), because of the change of the base year in the estimation of national accounts to that year.[7] The results are very similar, and it is recommended to use those obtained with the W methodology as well. Below, the optimal values of $\lambda$ calculated with each of the methodologies proposed by Marcet and Ravn (2003) are presented.

**Table 9. Estimations of optimal values for $\lambda$ with V and W methodology. 1991-2010**

| SERIES | V | W | Former |
|---|---|---|---|
| **IMAE** | 9800 | 29000 | 14087 |
| **Monthly GDP** | 6900 | 59000 | 14087 |
| **Quarterly GDP** | 1270 | 4020 | 1311 |
| **Yearly GDP** | 18 | 35 | 41 |

It can be concluded from this table that the behavior of the variations in the economic trend has changed when considering only this last period; the variability of the trend acceleration relative to the variability of the cycles has become more similar to the one observed in the American economy, while the variability of the trend acceleration has become less similar to the behavior in that economy (except for the yearly series, in both cases).

[7] Besides, it is important to highlight that for this period the official estimation that is used internally at the Central Bank of Costa Rica was used as the monthly GDP series.

As a reference, a graph with the trend's growth rates estimated for the case of the quarterly product series is shown below.

**Graph 5**
**Year-on-year variation rates of the Potential Output.**
**Quarterly series. Costa Rica 1992-2010**